Master Summer Refreshing Camp in Mathematics 2024

Dates 2 September 2024 - 13 September 2024
Place Campus Belval, University of Luxembourg; online option upon justified request
Target Groups 1st year students of the Master in Mathematics and the Master in Secondary Education – Mathematics at the University of Luxembourg
Open to other students and PhD candidates (also from other universities)
Registration By email to mmath@uni.lu

All first year students of the Master in Mathematics and the Master in Secondary Education - Mathematics are requested to participate in the Master Summer Refreshing Camp.
It is offered and designed especially for them:

- Master in Mathematics - General Mathematics (GM)
- Master in Mathematics - Financial Mathematics (FM)
- Master in Mathematics - Mathematical Modelling and Computational Sciences (MMCS)
- Master in Secondary Education - Mathematics (MSE-M)

The Refresher Courses are open to other students, including PhD candidates.

The aim of the Master Summer Refreshing Camp is to bring all students to the level required by the regular courses of the first semester. In particular, most prerequisites of the 1st semester courses will be recalled and practised with some exercises.

These Refresher courses help you recall subjects you learnt earlier. In case you never learnt some of these subjects, you must learn them by yourself before participating in the camp.

Week	Course	Duration	Compulsory for	Optional for	Lecturer
1	Analysis	8h	all		Guendalina Palmirotta
	Measure Theory	8h			Pierre Perruchaud
	Linear Algebra & Topology	8h			Augustinas Jacovskis
2	Analysis	8h	all		Guendalina Palmirotta
	Probability	8h	GM, FM	MSE-M, MMCS	Michele Stecconi
	Algebra	8h	GM, MSE-M	FM, MMCS	Augustinas Jacovskis

Course Content

Refresher Course in Analysis (16h)

- Basic real analysis of functions of one variable
- Basic complex analysis for functions in one variable:
  definition of holomorphic, power series development, identity theorem, Cauchy Riemann equations, complex path integration and use of the residue theorem for the computation of real integrals
- Basis real analysis of several variables:
  derivatives, inverse and implicit function theorems
- Ordinary Differential Equations:
  elementary solution techniques like separation of variables and variation of the constant, existence and uniqueness theorem (Picard/Lindelöf), some facts about linear ODE

Refresher Course in Measure and Integration Theory (8h)

- Basics of measure theory and Elementary probability: what is a measure, what is a sigma field, Borel sigma fields, Lebesgue measures, the definition of Lebesgue integral, limit theorems (Fatou Lemma, Dominated Convergence, Monotone convergence), change of variable formulae, product measures, product sigma fields, Fubini theorem)

Refresher Course in Probability (8h)

- Conditional expectation with respect to a sigma-field, basic notions of Hilbert spaces (L^2 space of a probability measure), projections, Conditional expectation as a projection.

Refresher Course in Linear Algebra and Basic Topology (8h)

- Linear algebra:
  vector spaces, linear maps, span and linear independence, eigenvalues and eigenvectors, bases on finite dimensional vector spaces, determinants and inverses, characteristic polynomials, inner products, orthonormal basis, Gram-Schmidt, norms on vector spaces
- Point-set topology:
  open sets, closed sets, basic operations on open and closed sets, continuous maps, connectedness, compactness, Hausdorff topologies

Refresher Course in Algebra (8h)

- Basic group theory:
  groups and subgroups, homomorphisms, normal subgroups and quotient groups, isomorphism theorems, abelian groups
- Basic ring theory:
  rings and subrings, homomorphisms, ideals and quotient rings, isomorphism theorems, domains and division rings
- Basic commutative ring theory:
  prime ideals and maximal ideals, PIDs, UFDs
- Basic field theory:
  field extensions, algebraic and transcendental elements, splitting fields, finite fields

Resources for Analysis Refresher Course

Guendalina Palmirotta

June 2024

As is usual in mathematics, **doing exercises**, is the best way to learn and understand material. Students are encouraged to attempt exercises from the textbooks/lecture notes/exercise sheets referenced below. All the mentioned resources from this document can be found and download in the OneDrive folder: Refresher in Analysis - Literature[1]
Password: RefresherAnalysis2024.

Disclaimer: The resources below cover much more material than there will be in the refresher course, and certain parts of the material below are not relevant to courses in the Masters programme at Luxembourg. Hence, to find the relevant parts of each of the materials below, at least for the refresher course, it is best to look at the contents of the refresher course listed below.

Analysis refresher course contents

- Basic real analysis: functions of one variable and several variables, derivatives, inverse and implicit function theorems
- Ordinary Differential Equations: elementary solution techniques like separation of variables and variation of the constant, existence and uniqueness theorem (Picard/Lindelöf), some facts about linear ODE
- Basic complex analysis (of one variable): definition of holomorphic, power series development, identity theorem, Cauchy Riemann equations, complex path integration and use of the residue theorem for the computation of real integrals

[1] https://uniluxembourg-my.sharepoint.com/:f:/g/personal/guendalina_palmirotta_uni_lu/Emkxs9_LSLpIrrLetxXE-5IBmMdKkcuXM6M-d6cQsEjQ1A?e=Jor5GW&CT=1718021963646&OR=OWA-NT-Mail&CID=a45cde1c-8199-573b-5f36-5c8c3d312e23

Textbooks

- *Advanced Calculus*, A Geometric View, James J. Callahan, Springer (2010).
- *Complex Analysis*, Third Edition, Undergraduate Texts in Mathematics, Joseph Bak and Donald J. Newman, Springer (2010).
- *Mathematical Analysis I*, Second Edition, Claudio Canuto and Anita Tabacco, Springer (2015).
- *Visual Complex Analysis*, Tristan Needham, Clarendon Press, Oxford (1997).
- *Understanding Analysis*, Second Edition, Undergraduate Texts in Mathematics, Stephen Abbott, Springer (2010).

Notes freely available online

- Analysis 3 (with solutions of the exercises) by Job Kuit
  https://math.uni-paderborn.de/ag/rg/team/kuit/analysis-3
- Complex Analysis Lecture Notes by Dan Romik:
  https://www.math.ucdavis.edu/~romik/data/uploads/notes/complex-analysis
- Complex Analysis with Applications by Jacob Shapiro:
  https://web.math.princeton.edu/~js129/PDFs/teaching/MAT330_spring_2023/MAT330_Lecture_Notes.pdf
- Ordinary Differential Equation by Alexander Grigorian:
  https://www.math.uni-bielefeld.de/~grigor/odelec2007.pdf

Online (youtube) courses

Check Steve Brunton's youtube channel: https://www.youtube.com/@Eigensteve/videos.

- Differential Equations and Dynamical Systems by Stve Brunton:
  https://www.youtube.com/watch?v=9fQkLQZe3u8&list=PLMrJAkhIeNNTYaOnVI3QpH7jgULnAmvPA
- Crash Course in Complex Analysis by Steve Brunton:
  https://www.youtube.com/watch?v=_mv0q7-WF4E&list=PLMrJAkhIeNNQBRslPb7I0yTnES981R8Cg

RESOURCES FOR THE REFRESHER COURSE IN MEASURE THEORY AND PROBABILITY

BY MICHELE STECCONI

1. Contents

The course is separated into two parts: Measure Theory and Probability. The first part is needed for the second part.

1.1. **Measure Theory.**

(1) *Measurable spaces and functions:* Sigma-algebras; Sigma-algebra generated by a family 2; Borel Sigma-algebra of $\mathbb{R}^d$.
(2) *Positive measures*: Infinite positive sums; Definition of measure; Continuity properties of positive measures; Uniqueness of a measure; Existence of measures.
(3) *Measurable functions*: Definition and examples.
(4) *Integral* : Simple functions; Integration; Almost surely; Monotone convergence, Fatous Lemma, Dominated convergence; Some consequences.

1.2. **Probability.**

(1) *Product measures*: Product sigma-algebra, Product measure; Fubini and Tonelli Theorems; Exercises on Fubini-Tonelli.
(2) *Probability spaces*: Change of variables; Random variables; Independence; Expectation; Variance and moments; Examples.
(3) *Conditioning*: General definition; Radon-Nikodym theorem; Properties of the conditional expectation; Conditioning as a projection; The effect of independence; Conditional densities
(4) *Convergence notions*: Definitons; Most famous limit theorems

2. Main resources

The Lecture notes of the course are available on Moodle. They are based on the books: [1] (mostly) and [2].

The suggested way to make the best of the **refresher** course is: read the notes; in doing that, try to do some of the proofs and exercises, expecially when they look particularly unfamiliar; look into one of the references (or even on wikipedia) when you are curious, when you don't manage to do a proof after enough attempts, when you want to see a more detailed discussion of a concept, when you need more definitions,...

Date: June 10, 2024.

References

[1] E. Çınlar. *Probability and Stochastics*. Graduate Texts in Mathematics. Springer New York, 2011. (Cited on p.1)
[2] R. M. Dudley. *Real Analysis and Probability*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 2 edition, 2002. (Cited on p.1)

3. Notes online

Disregard this notes if you manage to have access to one of the books.

(1) Measure Theory:
http://www.math.ucdavis.edu/~hunter/measure_theory/measure_theory.html
(2) Notes on Elementary Probability:
https://www3.nd.edu/~dgalvin1/30530/30530_S19/book/Lect_Prob_class.pdf

Resources for Linear Algebra Refresher Course

As is usual in mathematics,

!! doing exercises !!

is the best way to learn and understand material. Students are encouraged to attempt exercises from the textbooks/lecture notes/exercise sheets referenced below.

Disclaimer: The resources below cover much more material than there will be in the refresher course, and certain parts of the material below are not relevant to courses in the Masters programme at Luxembourg. Hence, to find the relevant parts of each of the materials below, at least for the refresher course, it is best to look at the contents of the refresher course listed below.

Linear Algebra refresher course contents

Linear algebra: vector spaces, linear maps, span and linear independence, eigenvalues and eigenvectors, bases on finite dimensional vector spaces, determinants and inverses, characteristic polynomials, inner products, orthonormal basis, Gram-Schmidt, norms on vector spaces

Textbooks

1. *Linear Algebra* by K. Hoffman and R. Kunze
2. *Advanced Linear Algebra* by S. Roman
3. *Elementary Linear Algebra* by H. Anton and C. Rorres

   (a more elementary and computation focused book; see this if you need a reminder on the basic concepts of linear algebra)

Notes freely available online

1. *Linear Algebra* by S. J. Wadsley (notes by D. Chua)

   https://dec41.user.srcf.net/notes/IB_M/linear_algebra.pdf
2. *Advanced Linear Algebra* by D. Surowski (notes by A. Mohammad)

   https://www.math.ksu.edu/~dbski/writings/Linear_Algebra.pdf

Other free online resources

1. Professor Macauley's YouTube channel

   https://www.youtube.com/playlist?list=PLwV-9DG53NDwKJIwF5sANj6Za7qZYywAq

   (Advanced Linear Algebra playlist)

Exercises

1. Exercises for *Linear Algebra* by S. J. Wadsley (typed by D. Chua)

   https://dec41.user.srcf.net/notes/IB_M/linear_algebra_eg.pdf

Resources for Algebra Refresher Course

As is usual in mathematics,

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is the best way to learn and understand material. Students are encouraged to attempt exercises from the textbooks/lecture notes/exercise sheets referenced below.

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Algebra refresher course contents

- Basic group theory: groups and subgroups, homomorphisms, normal subgroups and quotient groups, isomorphism theorems, abelian groups
- Basic ring theory: rings and subrings, homomorphisms, ideals and quotient rings, isomorphism theorems, domains and division rings
- Basic commutative ring theory: computing with polynomials, prime ideals and maximal ideals, PIDs, UFDs, function field, integrally closed domains.
- Basic field theory: field extensions, algebraic and transcendental elements, splitting fields, finite fields

Textbooks

1. *Abstract Algebra* by D. Dummit and R. Foote
2. *A First Course in Abstract Algebra* by J. Fraleigh
3. *Algebra* by S. Lang (more advanced but basic topics also covered; further reading)

Notes freely available online

1. *Introduction to Abstract Algebra* by A. Paulin (groups, ring and fields)

   https://math.berkeley.edu/~apaulin/AbstractAlgebra.pdf
2. *An undergraduate course in Abstract Algebra* by R. Howlett (rings and fields only)

   https://www.maths.usyd.edu.au/u/bobh/UoS/rfwhole.pdf
3. *Group Theory* by D. Kreher (groups and fields only)

   https://pages.mtu.edu/~kreher/ABOUTME/syllabus/GTN.pdf

Other free online resources

1. MathDoctorBob's YouTube channel

   https://www.youtube.com/watch?v=aENEYDFQnfA&list=PLF379B0552AD17780

   (Groups, rings and fields playlist)

2. Richard Borcherds' YouTube channel (more advanced algebra topics discussed; to be treated as further study)

   https://www.youtube.com/@richarde.borcherds7998

Exercises

1. Exercises for *Groups* by J. Goedecke (typed by D. Chua)

   https://dec41.user.srcf.net/notes/IA_M/groups_eg.pdf

2. Exercises for *Groups, Rings and Modules* by O. Randal-Williams (typed by D. Chua)

   https://dec41.user.srcf.net/notes/IB_L/groups_rings_and_modules_eg.pdf

3. Exercises for *Galois Theory* by C. Birkar (typed by D. Chua)

   https://dec41.user.srcf.net/notes/II_M/galois_theory_eg.pdf

Resources for Topology Refresher Course

As is usual in mathematics,

!! doing exercises !!

is the best way to learn and understand material. Students are encouraged to attempt exercises from the textbooks/lecture notes/exercise sheets referenced below.

Disclaimer: The resources below cover much more material than there will be in the refresher course, and certain parts of the material below are not relevant to courses in the Masters programme at Luxembourg. Hence, to find the relevant parts of each of the materials below, at least for the refresher course, it is best to look at the contents of the refresher course listed below.

Topology refresher course contents

Point-set topology: open sets, closed sets, basic operations on open and closed sets, continuous maps, connectedness, compactness, Hausdorff topologies

Textbooks

1. *Topology: A First Course* by J. Munkres
2. *Topology Without Tears* by S. A. Morris
3. *Algebraic Topology* by A. Hatcher (further reading)

Notes freely available online

1. *Introductory Point-set Topology* by A. Hatcher

   https://pi.math.cornell.edu/~hatcher/Top/TopNotes.pdf
2. *Topology* by A. Choudhury and R. Prakash

   http://individual.utoronto.ca/rishibhp/notes/Topology_Notes.pdf
3. *Metric and Topological Spaces* by J. Rasmussen (notes by D. Chua)

   https://dec41.user.srcf.net/notes/IB_E/metric_and_topological_spaces.pdf

Other free online resources

1. DanielChanMaths YouTube channel

   https://www.youtube.com/watch?v=b3EQQfi7xIc&list=PLgAugiET8rrJEL_3aovzCWK2LBSwvPsFd

   (Point-set Topology playlist)

Exercises

1. Exercises for *Metric and Topological Spaces* by J. Rasmussen (typed by D. Chua)

   https://dec41.user.srcf.net/notes/IB_E/metric_and_topological_spaces_eg.pdf