LIST OF THE PREREQUISITES AND THE CURRICULUM OF THE REFRESHER COURSES

This document presents the mathematical tools that must be mastered by a student applying to our Master in Data Science. In the event that an admitted student has gaps in his or her training with regard to the use of these tools, refresher courses (which are not compulsory) are organised at the beginning of the academic year in September, before the start of the academic semester. During these refresher courses, the teacher recalls the classical mathematical definitions and notations and gives exercises on which the student can apply these tools to perform calculations.

1. Basic linear algebra

- Do you know what **a vector** of $\mathbb{R}^p$ is and what the geometric interpretation of a (n, p)-**matrix** is?
- Do you master the **classical operations on matrices**: addition, multiplication, inverse and do you know what their interpretations are?
- Do you know what **the rank of a matrix** is? Are you able to apply a change of basis on a matrix and explain what it means?
- Do you know what an **inner product** in $\mathbb{R}^p$ is and what its interpretation is?
- Do you know what **the orthogonal projection** onto a linear subspace is?
- Are you able to give the expression of the (orthogonal) projection matrix onto the linear span of linearly independent vectors $v_1, \ldots, v_d$ (that *are not* necessarily orthonormal)?
- Do you know that symmetric matrices can be **diagonalized**? Would you be able to compute **their eigenvalues and their eigenvectors**?
- Would you be able to compute the square root of nonnegative symmetric matrix and its k-th power?
- Do you know what **the singular value decomposition** of a (n, p)-matrix A of rank r is? Do you know what the geometric interpretations of these values are?

2. Analysis

- Do you know what these **classical math symbols** are: $\sum$, $\prod$, $\circ$ (for $f \circ g$).
- Do you know these classical real valued functions on the line as well as their graphs? cos, sin, tan, arctan, exp, log, $x \mapsto x^\alpha$ with $\alpha \in \mathbb{R}$?
- What is **the domain** of these functions?
- Would you be able to calculate **the derivatives and primitives** of these functions?
- Do you know the rules to differentiate $f \circ g$, fg, f/g?
- Do you know how to study the **continuity and differentiability** of a real-valued function on the line? Do you know how to study the variations of a function and how to find its (local) **extrema**? Do you know how to find **its limits** at $\pm\infty$ (if any)? For example, would you be able to study the functions $f : x \mapsto xe^{-x}$ and $g : x \mapsto e^{-|x|}$ on $\mathbb{R}$ and draw their graphs (without using a computer)?
- Do you known how to check that **the integral** of a function converges at $+\infty$ or $-\infty$?
- Do you known how to calculate the integral of a continuous function on an interval by means of a primitive or **an integration by parts**? For example, would you be able to say if the following integrals are well-defined and calculate their values?

$$\int_0^1 x \log x \, dx, \quad \int_0^{+\infty} \frac{dx}{1+x^2}, \quad \int_0^1 \frac{dx}{\sqrt{x}} \quad \text{and} \quad \int_{-1}^1 \sqrt{1-|x|}\, dx.$$

- Do you know how to find the **Taylor expansion** of a function at a point? How to find the limit of a function at a point? For example, would you be able to calculate the following limits?

$$\lim_{x\to 0} \frac{\sin x}{x}, \quad \lim_{x\to 0} \frac{1-\cos x}{x}, \quad \lim_{x\to 0} \frac{1-\cos x}{x^2} \quad \text{and} \quad \lim_{x\to 0} \frac{\log(1+x)}{\sin x}.$$

- Do you know what a **real valued sequence** $(u_n)_{n\geqslant 1}$ is? Do you know what a **monotone/increasing/decreasing/bounded/converging** sequence is? Do you know some sufficient conditions that ensure that a limit exists?
- Do you know what it means when we say that a sequence of functions $(f_n)_{n\geqslant 1}$ **converges pointwise** to a function f? Do you know what **uniform convergence** is? For example would you be able to calculate the following (pointwise) limits?

$$\lim_{n\to+\infty} x^n \quad \text{and} \quad \lim_{n\to+\infty} \left(1-\frac{x}{n}\right)^n \text{ for a given } x \geqslant 0.$$

- Are you able to calculate these series? $\sum_{n=0}^{+\infty} x^n$ for $x \in (-1,1)$ and $\sum_{n=0}^{+\infty} (x^n/n!)$ for $x \in \mathbb{R}$.
- Do you know what an **open subset** of $\mathbb{R}$ (or $\mathbb{R}^n$) is? What a **closed subset** of $\mathbb{R}$ (or $\mathbb{R}^n$) is? What a **compact subset** of $\mathbb{R}$ ($\mathbb{R}^n$) is? For

example, can you draw the following subsets of $\mathbb{R}^2$ and say if they are open? Closed? Compact?

$$\left\{(x,y) \in \mathbb{R}^2,\ x \in [-1,1], 0 \leqslant y \leqslant 2x\right\}$$
$$\left\{(x,y) \in \mathbb{R}^2,\ x \in (-1,1), 0 < y < x^2+1\right\}$$
$$\left\{(x,y) \in \mathbb{R}^2,\ x \geqslant 0, y \geqslant 0, 2x-2 \leqslant y \leqslant x+1\right\}.$$

- Do you know what the **partial derivatives** (of order 1 and 2) of a smooth real-valued function on $\mathbb{R}^n$ are and how to calculate them? What is the Hessian matrix associated to such a function? For example, would you be able to calculate the partial derivatives of order 1 and 2 of $f : (x,y) \mapsto xy^2$ defined on $\mathbb{R}^2$? Would you be able to calculate its Hessian?