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Aging, Fertility and Macroeconomic Dynamics*

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Abstract

A tractable model with heterogeneous households is proposed to analyze the two-way interactions between demographic and macroeconomic variables. Total population and labor-market participation are both endogenous and affected by economic as well as demographic factors. We perform a quantitative exercise focusing on trend dynamics based on Japanese data. Our counterfactual analysis reveals the role of labor-market participation costs, sunk costs of raising newborns, and technology progress.

Keywords: Heterogeneous workers, Aging, Productivity, Labor markets.
JEL Class.: E20, J11, J13, J21.

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1 Introduction

Advanced economies face stagnating growth in per-capita income, with demographic changes, such as aging populations and declining fertility, playing a critical role. While technological progress and labor-market dynamics are essential drivers of growth, demographic factors often remain underexplored in macroeconomic analyses. Yet, these factors directly influence the size and productivity of the labor force, creating feedback loops between economic growth and demographic trends.

This paper introduces a novel macroeconomic model that explicitly links demographic dynamics with macroeconomic variables. Our framework incorporates households with heterogeneous members, where fertility and labor-market participation decisions are endogenously determined. These decisions are shaped by macroeconomic factors, such as wages and income streams, while also affecting aggregate productivity and economic outcomes.

We focus on three key questions: (1) To what extent do demographic factors, such as rising mortality due to aging and declining fertility, affect economic growth? (2) How do policy changes aimed at increasing labor market participation and fertility impact demographic trends and macroeconomic variables? (3) How does technological progress interact with macroeconomic variables, such as GDP and demographic dynamics? To address these questions, we calibrate our model to Japanese data from 1970 to 2019, capturing the country's unique demographic and economic trajectory. We then conduct counterfactual simulations to assess the impact of policies aimed at reducing labor-market participation costs, lowering the cost of raising children, and increasing productivity growth.

Our findings reveal several important insights. First, reducing labor market participation costs increases the number of workers while lowering GDP *per worker*, but it has a limited impact on population growth or overall GDP. Second, lowering the cost of raising children effectively mitigates population decline while reducing GDP *per capita*, but it does not affect GDP per worker. Last, in the most realistic scenario, higher productivity growth can stimulate population growth by increasing expected future wages, but also reduces the number of workers through selection effects, highlighting trade-offs between productivity and labor supply.

In the model, the birth rate results from the balance between the expected lifetime sum of labor-market income and a fixed sunk cost for the creation and raising of newborns. The dynamics of labor-market income are driven by the interplay between individual productivity distributions – based on a Pareto distribution – and the repeated payment of labor-market participation costs. Thus, fertility, labor-market participation, and the total number of workers are influenced by *both* demographic and macroeconomic factors. Additionally, due to the heterogeneity in productivity and the endogenous nature of labor-market participation and employment, labor-market and demographic variables endogenously affect the aggregate efficiency of the economy through worker selection effects.

Although our theoretical model is broadly applicable to economies at any stage of demographic transition and economic development, we use Japan as a case study for our quantitative analysis. Japan is well known for its rising mortality rate due to an aging population and declining fertility since the post-war period. As a result, despite various policy measures aimed at mitigating population decline by reducing the costs of raising children, the population has started to shrink in recent years.[1] Further, recent trends indicate stagnation in GDP per capita, despite increasing labor market participation driven by policies encouraging elderly and female workers, in particular, to remain employed or to participate respectively.

We calibrate parameter values and estimate exogenous trends – depreciation of human capital, declining mortality rate, cost of raising newborns, labor-market participation costs, and labor productivity – to match the observed growth rates of GDP per worker, population, labor-market participation rate, and birth rate in Japan from 1970 to 2019. Using our structural model, we then conduct counterfactual analysis for future periods to explore the key driving forces behind these trends.

The analysis of labor-market participation costs reveals significant effects on participation rates and GDP per worker, but mostly through the number of workers. Higher participation costs reduce labor-market participation, while lower costs increase it. However, population declines uniformly across scenarios, as changes in labor-market participation costs do not affect the balance between the value of human beings, defined as the expected sum of lifetime income and the cost of raising newborns. Lower costs lower GDP per worker by attracting less efficient participants, whereas higher costs select more efficient workers and raise GDP per worker. GDP per capita remains stable across scenarios, reflecting a balance between participation rates and productivity. The policy aimed at changing labor-market participation thus carries distributional effects, as it implies more but less efficient work, and shifts the burden of production on workers while keeping GDP per capita constant.

Reducing sunk costs for newborns is effective in mitigating population decline. However, the additional population growth primarily consists of non-working individuals, reducing the share of the working population. While GDP per worker remains unchanged, GDP per capita grows more slowly due to the increased economic burden of larger non-working population. This result highlights the trade-off between promoting population growth and maintaining economic efficiency and equality.

Finally, higher productivity growth increases GDP per worker, but may lead to stronger or weaker population decline, depending on the interaction between trend productivity growth and the exogenous driving forces affecting population growth. In the simplified case where infant

[1] These policies include childcare and education support, parental leave, financial and housing assistance, work-life balance initiatives, and fertility treatment subsidies.

mortality and death rates are constant, wealth effects lead to a decline in population. However, in more complex scenarios where human capital depreciation and death rates evolve over time following the recent trend, higher productivity growth may help mitigate population decline.

Our general conclusion from a comprehensive model that includes endogenous fertility and labor market participation is that demographic trends and potential policies aimed at curbing them have relatively little effect on macroeconomic efficiency. They mostly affect the number of workers or population, and thus affect GDP per worker or GDP per capita, but through workers or capita. The effects of technology progress on demographic variables are quite stronger: stimulating innovation and productivity growth can help counter population decline. These findings highlight the intricate interplay between demographic policies, labor-market dynamics, and technological progress in shaping the trajectory of aging economies.

While we assume perfect risk sharing within the population, our analysis highlights the potential conflicts arising from the distributional consequences of policy changes. Using Japan as an example, population decline can be mitigated through policies that reduce the costs of raising children. However, we show that such policies inevitably place a greater burden on workers. To alleviate potential disputes, policies aimed at increasing labor-market participation would be desirable to foster a more equitable society. These conflicts become particularly pronounced when technological progress slows down.

Literature. Our paper contributes to the literature in various respects. First, trying to explain the lost decade in Japan, Hayashi and Prescott (2002) show the critical explanatory power of an exogenous TFP process but do not link the latter to demographic factors. Our model provides an intuitive mapping between aging, fertility, and productivity, and builds a bridge between papers trying to explain the recent productivity slowdown and papers looking at the effects of aging.

Second, most recent overlapping generation (OLG) models such as Choukhmane, Coeurdacier, and Jin (2023), Nishiyama (2015), Kitao (2015), McGrattan and Prescott (2018), or Katagiri, Konishi, and Ueda (2020) look at the consequences of aging on the conduct of (potentially optimal) public policies, and disregard the potentially endogenous effects of aging on productivity. Some OLG models look at the issue of how aging might affect productivity, such as Fougère and Mérette (1999) or Bouzahzah, De la Croix, and Docquier (2002), but within relatively complex frameworks based on simulations. In contrast, our approach is highly tractable and can be understood by looking at a couple of equations. In addition, the above-cited papers develop endogenous growth models with human capital to account for the endogenous productivity effects of aging. In our model, productivity is endogenously affected by the number of workers, which depends on how large the total population is given both aging and endogenous fertility but also on the time-varying average productivity of workers, that results from selection effects through labor-market participation decisions. Indeed, Kotschy and Bloom (2023) show that aging can affect growth through changes in the working-age population, which is also affected by labor-market participation, and both margins should be carefully considered, as in our model.

Closer to our paper, Fernández-Villaverde, Ventura, and Yao (2023) recently show the importance of looking at GDP per worker to assess the dynamism of advanced economies instead of GDP per capita, which could be misleading. Just as Fernández-Villaverde, Ventura, and Yao (2023), our model reproduces the steady increase in the GDP per worker over time in Japan. However, our model differs in that demography and labor-market participation are endogenous and result from an investments in human capital and from the selection of workers into the labor market.

Our paper also partly overlaps with Cooley and Henriksen (2018), who show how aging changes the composition of the labor force and thus alters the productivity of labor, which may account for a substantial fraction – up to a quarter – of the observed slowdown in TFP growth. Relatedly, Kydland and Pretnar (2019), shows how an aging population leads to structural changes in the allocation of time to care of sick older people and leads to lower labor-market participation, which then reduces productivity and GDP per capita. Although our model works through very different channels than these two contributions, it also links demographic factors to labor-market participation and can thus be seen as an interesting complement. Its main interest, we believe, is the original two-way interaction between demographic and economic factors through endogenous fertility.

Finally, our paper contributes to the rising literature using macroeconomic models with heterogeneous agents.[2] Different from these simulation-based approaches, we track the distribution of worker productivity with summary statistics as in Ghironi and Melitz (2005) or Hamano and Zanetti (2017) among many others. The critical difference is that the heterogeneous-agent approach applies to households rather than firms though. Hence, our model provides a very tractable way of introducing households heterogeneity and its labor-market, demographic and macroeconomic implications. One key difference with respect to most heterogeneous-agents models currently used is, however, that we consider full risk-sharing among household members. While it greatly simplifies the model solution, it insulates aggregate saving decisions from the income risks associated with aging and declining fertility.

The paper is structured as follows. Section 2 presents a simple model of endogenous population and labor-market participation with heterogeneous workers that links demographic and economic factors in a novel and intuitive way. Section 3 discusses the calibration and explains how trends are estimated. Finally, Section 4 offers counterfactual experiments and Section 5 concludes.

[2] See Heathcote, Storesletten, and Violante (2009) for a survey including papers resorting to OLG models, and Kaplan and Violante (2018) for a more recent review, but focusing on heterogeneous-agents New Keynesian (HANK) models.

2 Model

We propose a model with endogenous household size and labor-market participation. Households supply labor monopolistically and receive firms' profits. Given the labor-market outcomes in terms of wages and the death rate of individuals, they can choose to increase or decrease the number of household members.

2.1 Firm and labor demand

Firms are perfectly competitive in goods market. The representative firm has the following aggregate production function:

$$Y_t = a_t H_t, \tag{1}$$

where a_t denotes the Total Factor Productivity (TFP hereafter), and h_t is a bundle of the different varieties of labor ω:

$$H_t = \left[\int_{\omega \in \Omega} z(\omega) \, h_t(\omega)^{\frac{\theta-1}{\theta}} \, d\omega \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $z(\omega)$ is the productivity of variety ω. The firm maximizes its profits, given that total labor expenditure are $\int_{\omega \in \Omega} w_t(\omega) \, h_t(\omega) \, d\omega$, which gives the following labor demand function for variety ω:

$$h_t(\omega) = \left(\frac{w_t(\omega)}{W_t z(\omega)} \right)^{-\theta} H_t, \tag{3}$$

where

$$W_t = \left[\int_{\omega \in \Omega} z(\omega) \left(\frac{w_t(\omega)}{z(\omega)} \right)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}. \tag{4}$$

We choose the price of the final good as numeraire.

2.2 Household sector

There is a unit continuum of households. In every period t, household j is made of two types of individuals: $m_t(j)$ individuals who are already members of the household at the beginning of the period, and $m_{et}(j)$ new individuals who join the household during the period (such as children). At the end of period t, a fraction $delta_t \in [0,1]$ of all existing individuals is subject to an exogenous mortality shock. Similarly, a fraction $tau_t \in [0,1]$ of all newborns faces an exogenous infant mortality shock. We assume that it takes one period for a newborn to become an 'active' member of the household. The total number of members in household (j) thus evolves according to:

$$m_{t+1}(j) = (1 - \delta_t) \, m_t(j) + (1 - \tau_t) \, m_{et}(j). \tag{5}$$

Among the $m_t(j)$ individuals in the household at the beginning of period t, only the most productive enter the labor market. Labor-market entry is subject to the repeated payment of a participation cost f_{nt}, also paid in units of labor. This cost can be thought to represent on-the-job training costs or various job-related types of expenditure like transport, specific clothing/equipments or commuting.

Consider a continuum of individuals with heterogeneous productivity that supply differentiated types of labor within the household. Household j allows for endogenous entry (fertility) and endogenous participation in the labor-market. Over the entire space of individuals, only a subset will actually work and choose to pay the fixed cost. Each individual draws a specific random labor quality z from a probability density function $\mu(z)$ upon entering the household. When she works, she supplies labor for a given amount of demand given by Equation (3).

The decision for total consumption and the creation of new members belongs to the household level while labor supply is made at individual level. Household j and each individual member with productivity z jointly maximize the following discounted sum of life time utility:

$$\max \mathbb{E}_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s(j)^{1-\sigma}}{1-\sigma} - \eta \frac{L_s(j)^{1+\varphi}}{1+\varphi} \right) \right\}, \tag{6}$$

where $C_t(j)$ and $L_t(j)$ respectively denote total consumption and hours worked in the household. Parameters σ and φ respectively stand for the constant degree of relative risk aversion and the inverse of Frisch elasticity on labor supply. Note also that $L_t(j)$ is defined as:

$$L_t(j) = \left[\int_{z_{\min}}^{\infty} z \ell_t(j,z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}.$$

Maximization of welfare is subject to the form of the bundle of labor and to the following budget constraint:

$$C_t(j) + x_{t+1}(j) v_t(j) (m_t(j) + m_{et}(j)) = x_t(j) m_t \left(v_t(j) + \widetilde{d}_t(j) \right). \tag{7}$$

In this equation, $m_t(j)$ and $m_{et}(j)$ have already been defined, $v_t(j)$ is the value of human capital and $\widetilde{d}_t(j) = \int_{z_{\min}}^{\infty} (w_t(j,z) \ell_t(j,z) - w_t f_{nt}) dM(z)$ denotes the average monopolistic profits made by workers, and $x_t(j)$ is the share of the mutual fund held by household (j). Optimization is also subject to the labor demand addressed to each individual with productivity z:

$$\ell_t(j,z) = \left(\frac{w_t(j,z)}{W_t(j) z} \right)^{-\theta} L_t(j). \tag{8}$$

First-order conditions (FOCs) with respect to $C_t(j)$, $x_{t+1}(j)$ and $w_t(j,z)$ give:[3]

$$
\begin{aligned}
C_t(j)^{-\sigma} &= \lambda_t(j), && (9)\\
\beta \frac{m_{t+1}(j)}{m_t(j)+m_{et}(j)} \mathbb{E}_t \left\{ \frac{\lambda_{t+1}(j)}{\lambda_t(j)} \frac{v_{t+1}(j)+\widetilde{d}_{t+1}(j)}{v_t(j)} \right\} &= 1, && (10)\\
\chi L_t(j)^{\varphi} &= W_t(j)\lambda_t(j), && (11)
\end{aligned}
$$

where $\chi = \eta\theta/(\theta-1)$ and where $\lambda_t(j)$ is the marginal utility of consumption. Then selection in the labor market takes place, and the threshold condition to work is given by:

$$
w_t(j, z_{nt}(j))\,\ell_t(j, z_{nt}(j)) = W_t(j) f_{nt}, \tag{12}
$$

where $z_{nt}(j)$ denotes the cut-off level of productivity above which household members choose to work. This equation states that the last household member entering the labor market is productive enough for his labor income to cover the entry cost. Finally, entry incurs a once and for all sunk costs f_{et} (education, childcare), paid in units of basket of workers as defined by Equation (2). As a consequence, the number of new family members is determined by the following free entry condition:

$$
v_t(j) = W_t(j) f_{et}. \tag{13}
$$

2.3 Aggregation

Individual-specific labor productivity z has a Pareto distribution with lower bound $z_{\min}$ and shape parameter $\varepsilon(1+\theta\varphi) > \theta - 1$, where θ is the elasticity of substitution across the different types of labor and φ the inverse of the Frisch elasticity of labor supply. The cumulative density function is $M(z) = 1 - (z_{\min}/z)^{\varepsilon}$. Let $\widetilde{z}_t(j)$ be the average productivity across household members and $\widetilde{z}_{nt}(j)$ the average productivity across workers, both defined as:[4]

$$
\widetilde{z}_t(j) = \left[\int_{z_{\min}}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}} dM(z) \right]^{\frac{1+\theta\varphi}{\theta-1}}, \quad \widetilde{z}_{nt}(j) = \left[\int_{z_{nt}(j)}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}} \frac{dM(z)}{1-M(z_{nt})} \right]^{\frac{1+\theta\varphi}{\theta-1}} \tag{14}
$$

Average productivities are defined as harmonic means of individual productivities weighted by the relative utility of hours worked.[5] Given the Pareto distribution defined previously, average

[3]The FOC with respect to $w_t(j,z)$ is given by:

$$
\frac{\eta\theta}{\theta-1} z \ell_t(j,z)^{-\frac{1}{\theta}} L_t(j)^{\varphi+\frac{1}{\theta}} = \lambda_t(j) w_t(j,z).
$$

and plugging the labor demand equation (8), we get $\chi L_t(j)^{\varphi} = W_t(j)\lambda_t(j)$.

[4]Proofs for aggregation results and procedures are given in Appendix A and B.

[5]Equivalently, we also have:

$$
\widetilde{z}_t^{-1} = \int_{z_{\min}}^{\infty} z^{-1} \left(\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} \right)^{1+\varphi} \mu(z)\, dz. \tag{15}
$$

productivities $\widetilde{z}_t(j)$ and $\widetilde{z}_{nt}(j)$ are given by:

$$\widetilde{z}_t(j) = \widetilde{z}(j) = \nabla z_{\min}, \quad \widetilde{z}_{nt}(j) = \nabla z_{nt}(j) \tag{16}$$

where $\nabla = \left(\frac{\varepsilon(1+\theta\varphi)}{\varepsilon(1+\theta\varphi)-(\theta-1)}\right)^{\frac{1+\theta\varphi}{\theta-1}}$. We express the variables using these average and define $w_t(j, \widetilde{z}_{nt}(j)) \equiv \widetilde{w}_{nt}(j)$ and $\ell_t(j, \widetilde{z}_{nt}(j)) \equiv \widetilde{\ell}_{nt}(j)$. With the average dividend of workers such that $d_t(j, \widetilde{z}_{nt}(j)) \equiv \widetilde{d}_{nt}(j) = \widetilde{w}_{nt}(j)\widetilde{\ell}_{nt}(j) - f_{nt}$, we can rewrite the cut-off condition (12) as:

$$\widetilde{w}_{nt}(j)\widetilde{\ell}_{nt}(j) = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} w_t f_{nt}. \tag{17}$$

Finally, once the threshold $z_{nt}(j)$ is known, the number of labor-market participants is simply given by $n_t(j) = (1 - M(z_{nt}(j)))\, m_t(j)$ which is rewritten with the Pareto distribution as:

$$\frac{n_t(j)}{m_t(j)} = \left(\frac{\nabla}{\widetilde{z}_{nt}(j)}\right)^{\varepsilon}. \tag{18}$$

Using the above notations, the average dividends across *all* individuals in household j is:

$$\widetilde{d}_t(j) = \frac{n_t(j)}{m_t(j)}\widetilde{d}_{nt}(j) \tag{19}$$

.

In addition, the labor supply for the average level of productivity is expressed as:

$$\chi\widetilde{\ell}_{nt}(j)^{\varphi} = \widetilde{w}_{nt}(j)\,\lambda_t(j). \tag{20}$$

Finally, the wage index and the basket of hours worked are:

$$W_t(j) = (n_t(j)\,\widetilde{z}_{nt}(j))^{\frac{1}{1-\theta}}\,\widetilde{w}_{nt}(j)\,/\widetilde{z}_{nt}(j) \text{ and } L_t(j) = (n_t(j)\,\widetilde{z}_{nt}(j))^{\frac{\theta}{\theta-1}}\,\widetilde{\ell}_{nt}(j). \tag{21}$$

2.4 General equilibrium

In equilibrium, all households are homogeneous and we drop household index j. Perfect competition in goods market implies that $W_t = a_t$. The goods market clearing writes:[6]

$$Y_t = C_t. \tag{22}$$

The labor market clearing condition writes:

$$L_t = \frac{Y_t}{a_t} + n_t f_{nt} + m_{et} f_{et}. \tag{23}$$

[6]The same condition can be derived by aggregating the budget constraint (7) across individuals.

Finally, we define the total factor productivity and GDP as:

$$TFP_t = a_t \left(n_t \widetilde{z}_{nt}\right)^{\frac{\theta}{\theta-1}}, \tag{24}$$

and

$$GDP_t = W_t L_t. \tag{25}$$

A summary of the model's equilibrium condition and a derivation of the balanced-growth steady-state conditions are given in Appendix C and D respectively.

3 Analytical results

The original model presented above is highly non-linear and impossible to analyze without relying on numerical methods. However, we can gain intuition from looking at some analytical results that can be derived under specific assumptions, *i.e.* with a constant death rate and infant mortality rate, $g_\delta = g_\tau = 1$, where population dynamics simplifies.

Proposition 1 (Population and GDP per capita)**.** *Under $g_\delta = g_\tau = 1$, population m grows over time ($g_m > 1$) when:*

$$g_{f_e} g_a^{\frac{\sigma-1}{\sigma+\varphi}} g_\chi^{\frac{1}{\sigma+\varphi}} < 1, \tag{26}$$

and GDP per capita grows over time ($g_{GDP/m} > 1$) when:

$$g_a g_{f_e} > 1. \tag{27}$$

Proof. See Appendix F.

Start with the simple case where $g_{f_e} = g_\chi = 1$, which corresponds to the standard neoclassical growth model with human capital accumulation.

With only technological improvement ($g_a > 1$) and the standard parameterization such that $\sigma > 1$, total labor supply L decreases over time because of the wealth effect on labor supply driven by improvements in technology. Accordingly, population m declines steadily. Put differently, with economic growth, the sunk costs for newborns are increasing along with the increase in real wages even with $g_{f_e} = 1$. The increase in costs is larger than the rise in the value of individual human being (v) brought by a higher expected income due to the technological improvements. As a result, newborns and thus populations must fall in equilibrium.

As expected, a decline in costs of raising newborns implying $g_{f_e} < 1$ contributes to population growth. However, if the decline is too large, there is a risk for the economy to fall in a Malthusian trap characterized by a stagnating or declining level of GDP per capita ($g_{GDP/m} \leq 0$). In addition, a higher growth rate of the disutility of supplying labor ($g_\chi > 1$) works as a negative shock to labor supply, which in turn reduces total population. Further, when labor supply is completely inelastic ($\varphi = \infty$) as in the Solow model, the growth rate of productivity does not affect population growth. In this case, despite the improvement in technology, the labor supply does not fall and, as a result, only g_{f_e} matters for population growth. Finally, note that under our simplified assumptions, reforms on the labor market leading to changes (f_n) have no influence on population growth, leading to the following corollary.

Corollary 1 (Neutrality of labor market reforms)**.** *Assuming $g_\delta = g_\tau = 1$, changes in the cost of labor market participation (f_n) do not affect population growth.*

Intuition is the following. All else equal, when labor market participation (n/m) increases, it simultaneously reduces the average returns from human capital investments ($\tilde{d}_n$). The interaction of the two opposing forces keep the value of life v constant.

A second proposition characterizes the growth path of labor-market participation rate and GDP per worker under our simplified assumptions.

Proposition 2 (Labor market participation rate and GDP per worker)**.** *Under $g_\delta = g_\tau = 1$, the labor market participation rate n/m grows over time when:*

$$g_{f_e} > g_{fn}, \tag{28}$$

and GDP per worker grows when:

$$g_a g_{f_n} > 1. \tag{29}$$

Proof. See Appendix G.

When the cost of raising newborns g_{f_e} grows faster than the cost of participating in labor market g_{f_n}, population increases at a lower rate than the number of workers, which raises the labor-market participation rate (n/m) over time. Put differently, when this is the case, competition among potential workers in the labor market is not harsh and serves to increase labor market participation. A similar intuition governs the second part of Proposition 2: for GDP per worker to increase, productivity (which increases GDP) needs to grow more than participation costs (that raise the number of workers).

Specifically, Proposition 1 and Proposition 2 indicate that a labor-market reform changing the participation costs has no impact in improving GDP per capita since it is solely pinned down by g_a and g_{f_e}. Intuitively, when the participation increases, it necessarily provides incentive for less efficient members to work, and thus the overall impact on GDP per capita is null. However, labor-market reforms changing the participation cost induce distributional effects, as they determine who works and who does not for the entire member of the society.

Finally, when the death rate δ_t or the infant mortality rate τ_t are no longer constant, the above propositions are influenced by the impact of these two variables in general equilibrium.

Their effects run through population dynamics and alter the path of aggregate macroeconomic variables. It thus becomes impossible to get an analytical characterization of these interaction and we have to rely on numerical solutions and simulations, as we do in the next sections.

4 Calibration and estimation

We calibrate the parameters of our model to fit post-war Japanese data. Specifically, we define four sub-periods capturing distinct trends from 1970 to 2019, in line with Japan's economic and demographic shifts:

1. The first period, from 1970 to 1979, is characterized by a steady decline in the death rate.
2. The second period, from 1979 to 1991, is characterized by strong GDP growth, and aligns with the mortgage bubble in Japan's history, after which stagnation in both GDP and per capita GDP growth was observed.
3. The third period, from 1991 to 2008, features continuous population growth, building on previous trends.
4. The fourth period, from 2008 to 2019, is characterized by stagnating and then declining population growth.

We assume that the discount factor (β), Frisch elasticity of labor supply, risk aversion (σ), the elasticity of substitution between labor varieties (θ), and the Pareto shape parameter (ε) remain constant over time and identical across the four periods.

The time unit is a quarter so that $\beta = 0.99$ implies an annual real interest rate of 4%. We restrict the utility function so that $\sigma = 1.5$, assume a Frisch elasticity of $\varphi^{-1} = 0.3333$ and an elasticity of substitution among workers with $\theta = 4$. These values are in line with those used in the literature such as Fujiwara et al. (2005) and Sugo and Ueda (2008). Kitao and Yamada (2019) document the evolution of income dispersion in Japan in the post-war period using data from the National Survey of Family Income and Expenditure. Accordingly, we set the Pareto wage dispersion parameter to $\varepsilon = 1.1409$ to replicate the Survey values (see Appendix H for the detailed mapping between income dispersion and our parameter ε). These parameter values are summarized in Table 1 below.[7]

We calibrate the growth rate of the death rate (g_δ) using directly observed data. The disutility of supplying labor (χ, and consequently η, since $\chi = \eta\theta/\,(\theta - 1)$) is assumed to remain constant within each period (i.e., $g_\chi = 1$) but its level is adjusted.[8]

[7] We solve the model using the perturbation method with the RISE toolbox in MATLAB, as proposed by Maih (2015).

[8] This assumption allows for a declining intensive margin of labor supply, consistent with observations in Japanese data.

Table 1: Calibration

β	Discount factor	0.99
φ	Inverse of elasticity of labor supply	3
σ	Risk aversion	1.5
θ	Elasticity of substitution among workers	4
ϵ	Pareto shape	1.1409
f_e	Sunk costs for newborns (initial level)	1
a	Technology (initial level)	1

We then estimate the growth rates of four parameters: the depreciation rate of newborns (g_τ), technology (g_a), labor market participation costs (g_{f_n}), and sunk costs in creating newborns (g_{f_e}), to match the observed growth rates of (a) population, (b) the employment rate, (c) GDP per worker, and (d) GDP per capita for each time period defined above. Appendix E provides some details about our calibration procedure and Table 2 summarizes the estimation results.

Table 2: Estimation

		1970-1979	1979-1991	1991-2008	2008-2019
Data					
g_δ	Growth rate of death rate	0.9861	1.0085	1.0172	1.0175
Adjusted					
χ	Disutility in supplying labor	2.0817	2.1479	5.0095	5.9118
Estimated					
g_τ	Growth rate of Infant mortality rate	0.9862	0.9790	0.9929	0.9210
g_a	Growth rate of technology	1.0540	1.0524	1.0125	1.0098
g_{f_n}	Growth rate of labor-market cost	0.9846	0.9794	0.9973	0.9911
g_{f_e}	Growth rate of costs of raising newborns	1.0106	1.0160	1.0033	1.0168

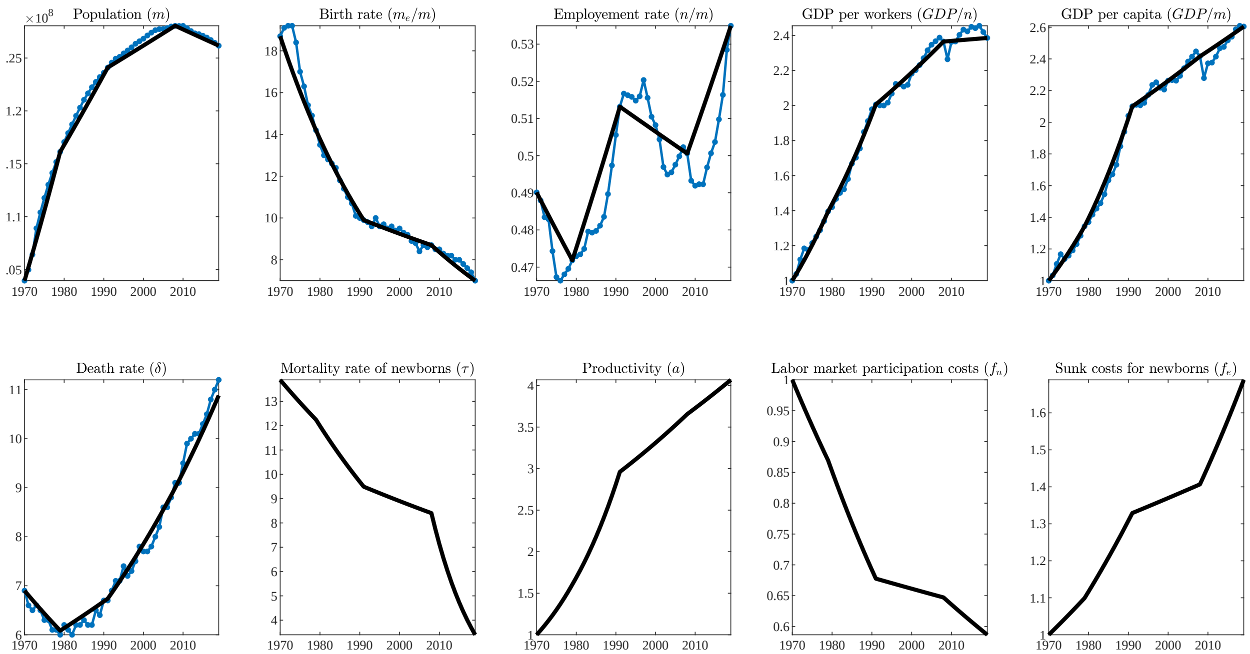
5 Counterfactual experiments

We simulate the model using the parameter values presented in the previous section. Specifically, the steady state with growth is computed for each time period, from which we perform the perfect-foresight simulation. After demonstrating the model's fit with the data, we present future projections of Japanese demographics and economic performance under different scenarios involving technology, labor market reforms, and sunk costs for newborns.

5.1 Data vs. model

We start our analysis with a comparison between the theoretical model and the data. As illustrated in Figure 1, the theoretical model fits the post-war Japanese data quite well. Notably, the model accurately captures the trend dynamics of not only the targeted variables but also non-targeted GDP per capita.

Figure 1: Data vs. Model



To replicate these dynamics, the model requires a secular decline in the depreciation rate of newborns (τ) and labor-market participation costs (f_n), along with a secular increase in the sunk entry costs for newborns (f_e) and technology (a) over the sample period. Importantly, the growth rate of productivity becomes smaller after the collapse of the bubble economy in 1991.

Building on this benchmark simulation, we then proceed to conduct a counterfactual analysis for the next 20 years (until around 2040). This analysis examines hypothetical scenarios involving the growth rates of the calibrated and estimated parameters: g_a, g_{f_n}, and g_{f_e}.

These future scenarios are presented under different assumptions about human capital depreciation rates. The first assumption is that these rates remain constant after 2019, *i.e.*, $g_\tau = g_\delta = 1$, just as in Section 3. A more general case assumes that the growth rate of the depreciation rate of newborns increases by 5 percentage points compared to the last period, while the growth rate of the death rate decreases by 0.7 percentage points compared to the last period. These values correspond to estimates of the future trajectories of infant mortality and death rates in Japan.[9]

[9] The parameter values align with forecasts by the National Institute of Population and Social Security Research. Their projections suggest that the number of deaths will peak in 2043, while the death rate will peak later, in 2065, at 17.7, due to continued aging.

5.2 Labor-market participation costs

Figure 2 illustrates three different scenarios for the Japanese economy under varying growth rates of the fixed costs for labor-market participation (g_{f_n}). The benchmark case, represented by solid lines, assumes that the growth rates of exogenous parameters in the future period follow the same trend as in the previous period. In contrast, we consider two alternative scenarios: one where the growth rate of f_n is one percentage point higher (dashed grey) and another where it is one percentage point lower (dashed light grey) compared to the benchmark scenario.

We primarily focus on the case where $g_\tau = g_\delta = 1$ (panel (a)) to illustrate the basic intuition of the model. As anticipated from Proposition 2 and the previous sections, changes in g_{f_n} have no impact on population growth when $g_\tau = g_\delta = 1$. Since the number of newborns falls proportionally, the birth rate (m_e/m) is constant over time.

Accordingly, the labor-market participation rate (n/m) decreases over time when costs are higher, while it increases over time when costs are lower, relative to the benchmark case. Since the population (m) continues to decline at the same rate across all scenarios, the changes in labor-market participation are solely driven by variations in the number of workers (n). GDP and GDP per capita (GDP/m) follow the same trajectory across all scenarios.

As a result, GDP per worker (GDP/n) undershoots its baseline trajectory in the lower-cost scenario, while it increases in the higher-cost scenario. This occurs because an increase in labor-market participation involves the entry of less efficient workers, leading to a decline in average productivity, as reflected in GDP per worker.

In a more general case (panel (b) of Figure 2), where the death rate is increasing and the depreciation rate of newborns is further decreasing ($g_\tau < 1$ and $g_\delta > 1$), the dynamics with respect to changes in the growth rate of labor market participation costs are shown to be qualitatively similar.

5.3 Sunk Costs for Newborns

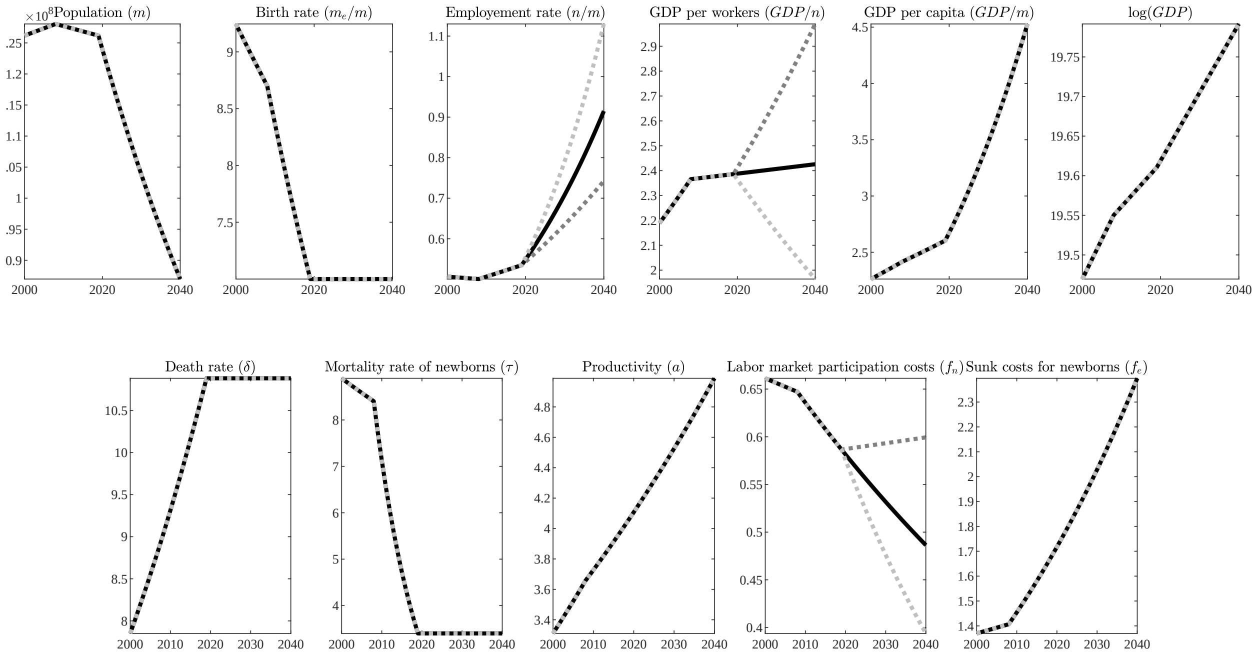
Next, we present future scenarios focusing on the sunk costs of raising newborns (f_e). In the benchmark case, the growth rate of exogenous parameters is again assumed to follow the same path as in the previous period (solid lines in Figure 3). For the alternative scenarios, we consider cases where the growth rate of f_e is higher by one percentage point (dashed grey) and lower by one percentage point (dashed light grey).

As shown in Figure 3 (panel (a)), lower costs of raising newborns effectively mitigate the decline in population (m). In this scenario, the population decline in the future periods is less pronounced compared to the other cases.

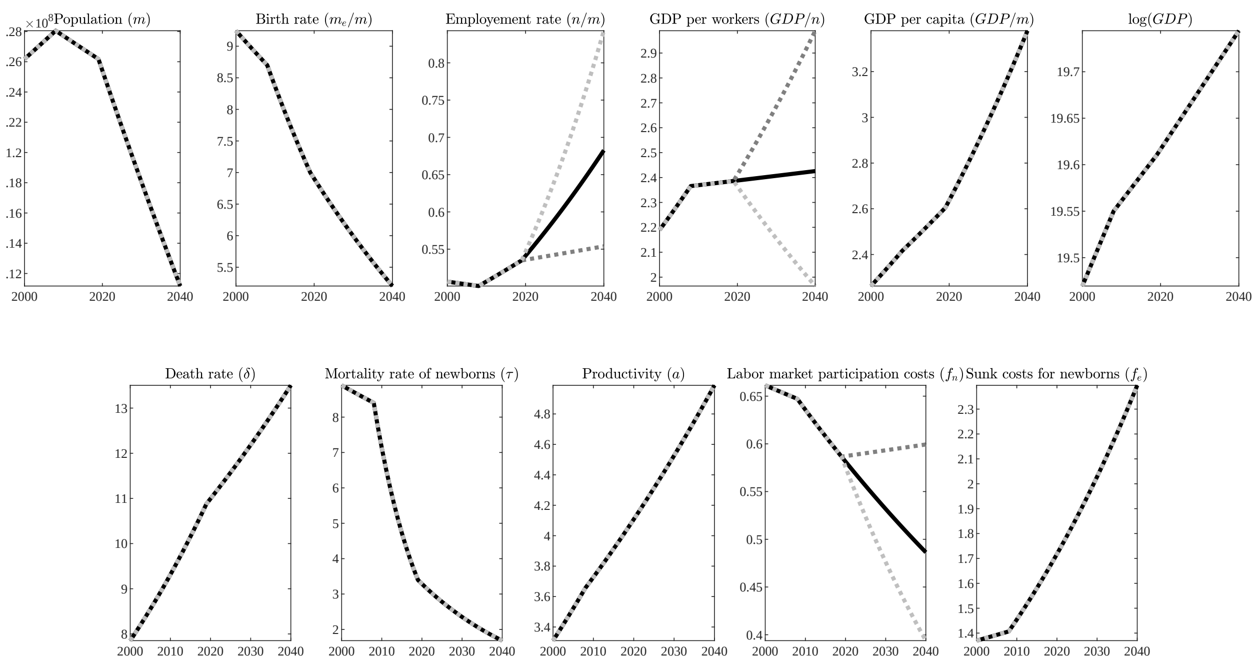
However, the participation rate (n/m) decreases relative to the other scenarios, indicating that the additional population growth primarily consists of individuals that do not participate in the

Figure 2: Labor-market participation costs

(a) $g_\tau = g_\delta = 1$



(b) $g_\tau < 1$ and $g_\delta > 1$



Solid black: baseline. Dashed grey: larger increase in labor-market participation cost. Dashed light grey: larger decrease in labor-market participation cost.

labor market. The working population remains as productive as in other scenarios since the Gross Domestic Product (GDP) per worker (GDP/n) follows the same growth trajectory irrespective of the different paths of the cost of raising newborns.

However, GDP per capita (GDP/m) grows less when total population declines more mildly, compared to the other simulations. This occurs because a relatively smaller working population must support a larger non-working population.

The question of whether higher birth rates are a viable solution for aging economies like Japan remains a subject of public debate. Our simulation results help inform this debate and suggest that increasing the birth rate by reducing sunk costs for newborns can indeed stimulate population growth. However, this comes at the cost of a reduction in GDP per capita, assuming all else remains equal.

These dynamics uncover an inherent tension between the working and non-working segments of the population. However, exploring these socio-economic tensions falls outside the scope of our current analysis.

In a more general case (panel (b)), where the death rate is increasing and the depreciation rate of newborns is further decreasing ($g_\tau < 1$ and $g_\delta > 1$), the dynamics with respect to changes in the growth rate of sunk costs for newborns are shown to be qualitatively similar.

5.4 Technology

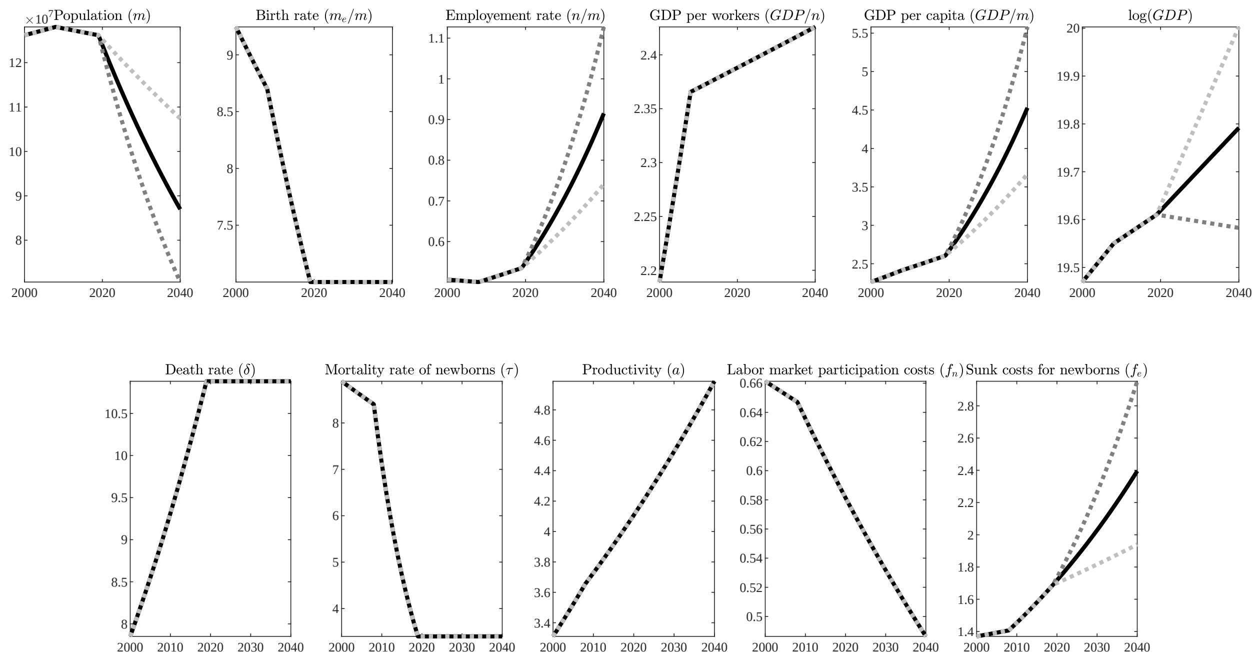
Figure 4 now presents different scenarios for the future Japanese economy under three alternative growth rates of labor productivity (g_a). The benchmark scenario, shown with solid lines, assumes that technological growth continues to follow the same trend as in the previous period. Dashed grey lines represent the scenario with a one percentage point higher growth rate of labor productivity compared to the benchmark, while dashed light grey lines correspond to a one percentage point lower growth rate.

When $g_\tau = g_\delta = 1$, the special case studied in Section 3, Proposition 1 concluded that population declined further under a higher growth rate of labor productivity due to the wealth effect on labor supply. This is confirmed looking at Figure 4 (panel (a)). Furthermore, consistent with both Proposition 1 and Proposition 2, changes in the technological growth rate are neutral with respect to the birth rate (m_e/m) and the employment rate (n/m). However, a higher growth rate of technology leads to improvements in both GDP per working-age population (GDP/n) and GDP per capita (GDP/m).

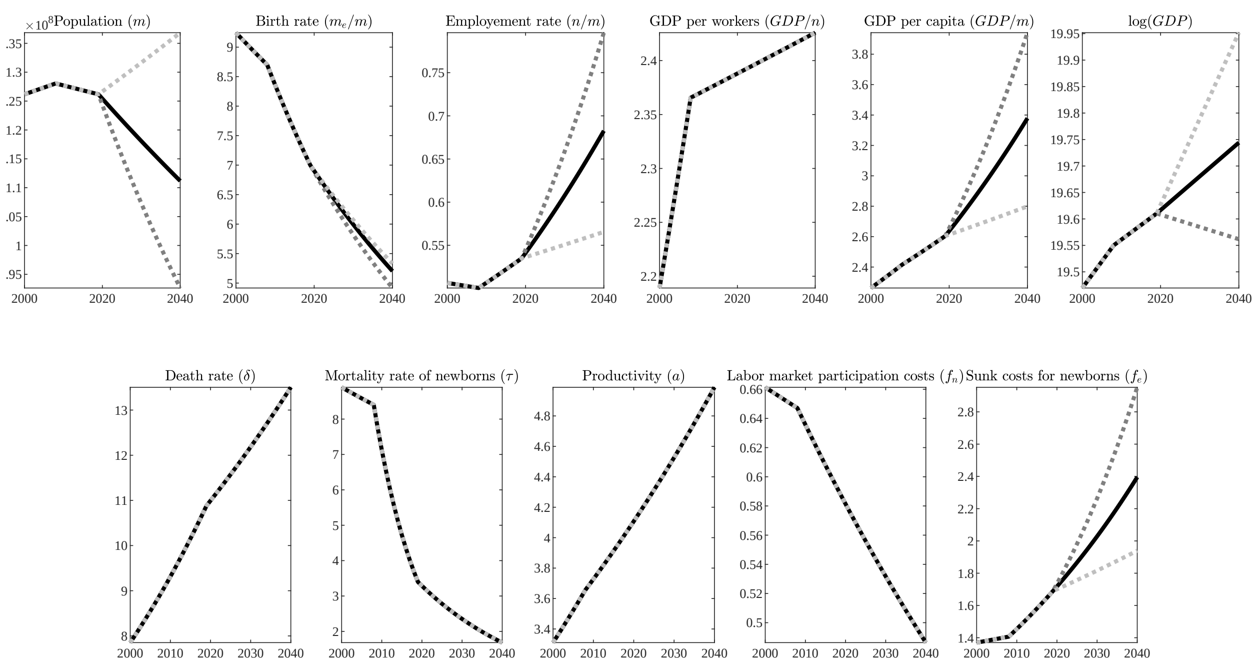
In the more general scenario where the growth rate of human capital depreciation satisfies $g_\tau < 1$ and $g_\delta > 1$ (panel (b) in Figure 4), the model's non-linearity becomes more pronounced. Interestingly, under these conditions, higher technological growth can mitigate population decline.

Figure 3: Sunk costs of raising newborns

(a) $g_\tau = g_\delta = 1$



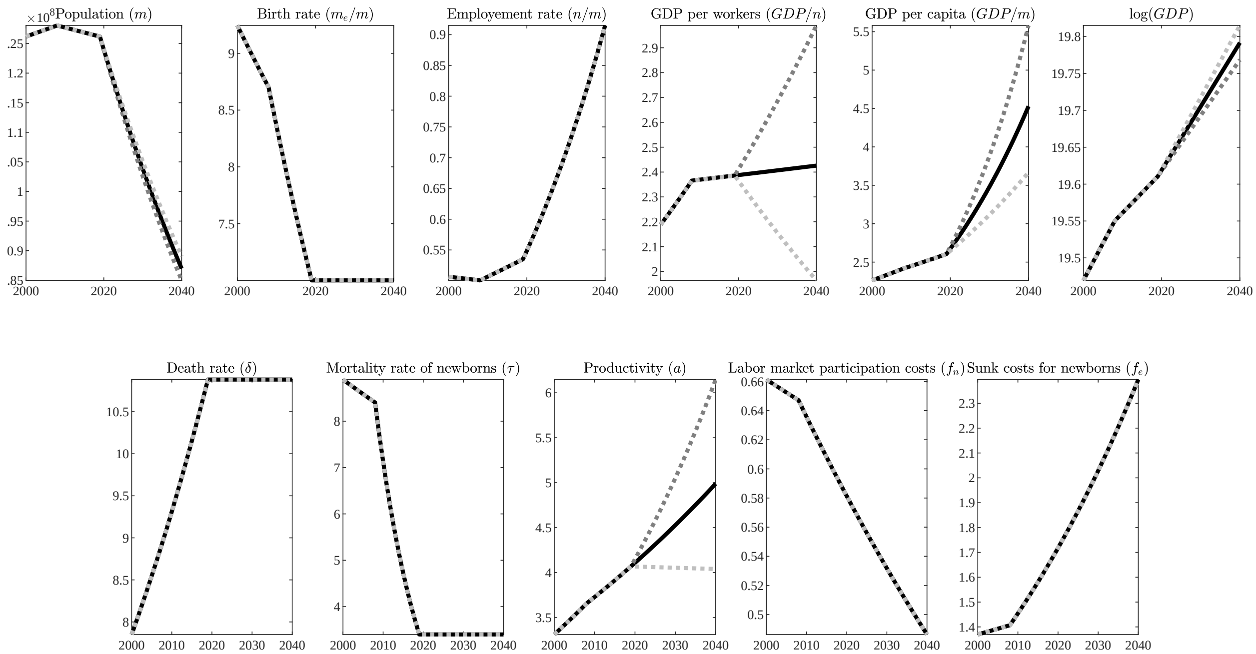
(b) $g_\tau < 1$ and $g_\delta > 1$



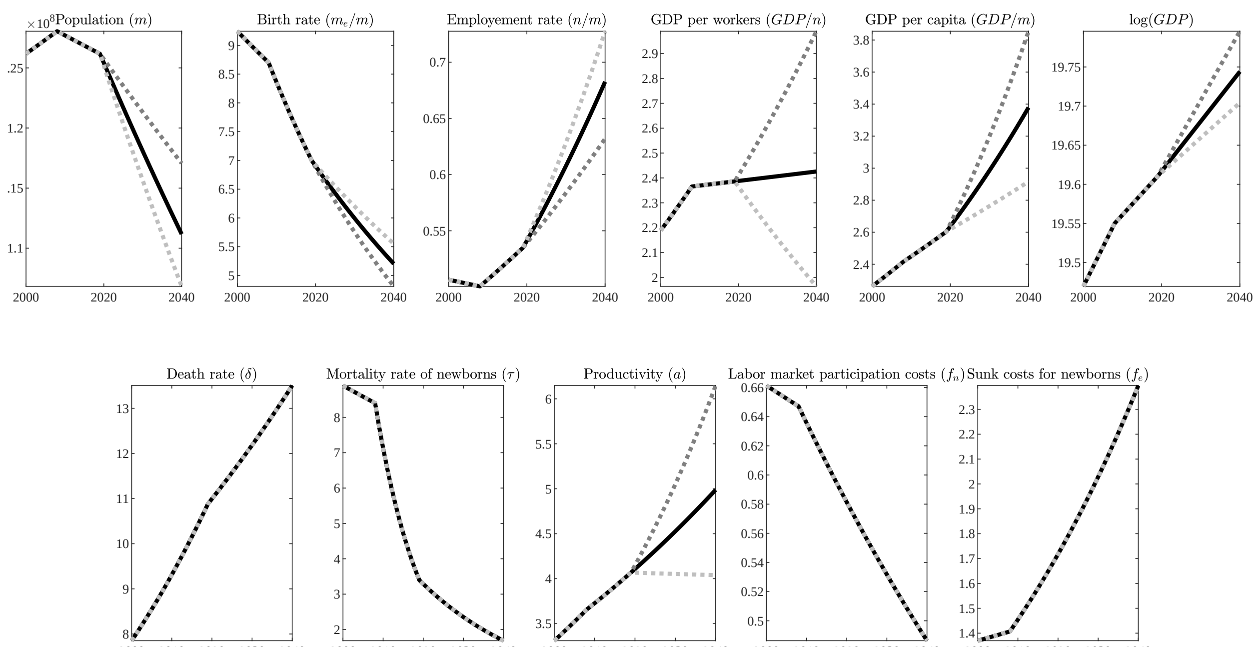
Solid black: baseline. Dashed grey: larger increase in cost of raising newborns. Dashed light grey: smaller increase in cost of raising newborns.

Figure 4: Productivity

(a) $g_\tau = g_\delta = 1$



(b) $g_\tau < 1$ and $g_\delta > 1$



Solid black: baseline. Dotted grey: larger increase in productivity. Dotted light grey: small decline in productivity.

6 Conclusion

In this paper, we investigate the relationship between demographic trends and economic variables. Our analysis is based on a theoretical model that aligns with historical Japanese data from the 1970s onward. The model highlights the critical role of demographic factors in shaping economic trends, particularly through labor-market participation costs and the sunk costs of raising children. These policy measures operate by influencing the number of individuals or workers, thereby affecting macroeconomic aggregates.

The model also shows that changes in productivity directly affect both macroeconomic and demographic variables in general equilibrium. In the most realistic scenario, faster productivity growth can help mitigate population decline while increasing both GDP per capita and GDP per worker. This suggests that policies aimed at fostering innovation can complement those designed to curb population decline in aging economies.

Although our model does not explicitly address distributional consequences or inequality among household members, it implicitly encompasses these aspects by assuming perfect consumption risk sharing among productive (working) households and non-productive households. Looking at the distributional consequences of aging, declining fertility and labor-market participation within our model is an interesting topic on its own that we leave for future research.

In addition, the role of migration, which can be considered isomorphic to the arrival of newborns with different nationalities and potentially higher social costs, emerges as another compelling direction for future research. Investigating this dimension could further enrich our understanding of the demographic and economic interplay, especially in the context of an increasingly globalized world.

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A Proof for aggregation

Let $\widetilde{z}_t$ denote the average productivity of households, such that

$$W_t = (m_t\widetilde{z}_t)^{\frac{1}{1-\theta}} w_t(\widetilde{z}_t)/\widetilde{z}_t \tag{A.1}$$

$$L_t = (m_t\widetilde{z}_t)^{\frac{\theta}{\theta-1}} \ell_t(\widetilde{z}_t) \tag{A.2}$$

The labor supply condition in relative terms writes

$$\left(\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)}\right)^{\varphi} = \frac{w_t(z)}{w_t(\widetilde{z}_t)} \tag{A.3}$$

and the demand condition in relative terms writes

$$\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} = \left(\frac{w_t(z)/z}{w_t(\widetilde{z}_t)/\widetilde{z}_t}\right)^{-\theta} \tag{A.4}$$

Combining gives:

$$\frac{w_t(z)}{w_t(\widetilde{z}_t)} = \left(\frac{z}{\widetilde{z}_t}\right)^{\frac{\theta\varphi}{1+\theta\varphi}} \tag{A.5}$$

and

$$\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)} = \left(\frac{z}{\widetilde{z}_t}\right)^{\frac{\theta}{1+\theta\varphi}} \tag{A.6}$$

Plugging the above condition into the wage index expressed on the space of workers z (of mass m_t) — instead of the space of labor types ω — then gives

$$W_t = \left[\int_{z_{\min}}^{\infty} z\left(\frac{w_t(z)}{z}\right)^{1-\theta} m_t\mu(z)\,dz\right]^{\frac{1}{1-\theta}} \tag{A.7}$$

$$= \underbrace{(m_t\widetilde{z}_t)^{\frac{1}{1-\theta}} w_t(\widetilde{z}_t)/\widetilde{z}_t}_{W_t}\widetilde{z}_t^{\frac{1}{1+\theta\varphi}}\left[\int_{z_{\min}}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}}\mu(z)\,dz\right]^{\frac{1}{1-\theta}} \tag{A.8}$$

which then implies

$$\widetilde{z}_t = \left[\int_{z_{\min}}^{\infty} z^{\frac{\theta-1}{1+\theta\varphi}}\mu(z)\,dz\right]^{\frac{1+\theta\varphi}{\theta-1}} \tag{A.9}$$

or equivalently:

$$\widetilde{z}_t^{-1} = \int_{z_{\min}}^{\infty} z^{-1}\left(\frac{\ell_t(z)}{\ell_t(\widetilde{z}_t)}\right)^{1+\varphi}\mu(z)\,dz \tag{A.10}$$

B Aggregation

Here we show that the aggregate budget constraint is equivalent to the labor market clearing. Aggregating the budget constraint across different households,

$$C_t + v_t\left(m_t + m_{et}\right) = m_t\left(v_t + \widetilde{d}_t\right) \tag{B.1}$$

Plugging the expression of $\widetilde{d}_t$,

$$C_t + v_t m_{et} = n_t \widetilde{d}_{nt} \tag{B.2}$$

Plugging the expression of $\widetilde{d}_{nt}$,

$$C_t + v_t m_{et} = n_t\left(\widetilde{w}_{nt}\widetilde{\ell}_{nt} - W_t f_{nt}\right) \tag{B.3}$$

We have $n_t \widetilde{w}_{nt}\widetilde{\ell}_{nt} = W_t L_t$, so

$$C_t + v_t m_{et} = W_t L_t - n_t W_t f_{nt} \tag{B.4}$$

With $Y_t = C_t$ and $v_t = W_t f_{et}$

$$Y_t + W_t f_{et} m_{et} = W_t L_t - n_t W_t f_{nt} \tag{B.5}$$

which, divided by W_t gives

$$\frac{Y_t}{W_t} + f_{et} m_{et} = L_t - n_t f_{nt} \tag{B.6}$$

with $W_t = a_t$ and rearranging,

$$L_t = \frac{Y_t}{a_t} + n_t f_{nt} + m_{et} f_{et} \tag{B.7}$$

C Model summary and reduction

The model boils down to:

$$\begin{aligned}
\text{Motion} &: \quad m_{t+1} = (1-\delta_t)\, m_t + (1-\tau_t)\, m_{et} && \text{(C.1)}\\
\text{Labor market clearing} &: \quad L_t = Y_t/a_t + n_t f_{nt} + m_{et} f_{et} && \text{(C.2)}\\
\text{Wage} &: \quad W_t = a_t && \text{(C.3)}\\
\text{Participation} &: \quad \frac{n_t}{m_t} = \left(\frac{\nabla}{\widetilde{z}_{nt}}\right)^{\varepsilon} && \text{(C.4)}\\
\text{ZCP} &: \quad \widetilde{w}_{nt}\widetilde{\ell}_{nt} = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} W_t f_{nt} && \text{(C.5)}\\
\text{Av. dividends} &: \quad \widetilde{d}_t = \frac{n_t}{m_t}\widetilde{d}_{nt} && \text{(C.6)}\\
\text{Av. dividends of workers} &: \quad \widetilde{d}_{nt} = \widetilde{w}_{nt}\widetilde{\ell}_{nt} - W_t f_{nt} && \text{(C.7)}\\
\text{Free entry} &: \quad v_t = W_t f_{et} && \text{(C.8)}\\
\text{Euler share} &: \quad \beta \frac{m_{t+1}}{m_t + m_{et}} E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{v_{t+1} + \widetilde{d}_{t+1}}{v_t} \right\} = 1 && \text{(C.9)}\\
\text{Goods market clearing} &: \quad C_t = Y_t && \text{(C.10)}\\
\text{Labor supply} &: \quad \chi L_t^{\varphi} = W_t \lambda_t && \text{(C.11)}\\
\text{Wage index} &: \quad W_t = (n_t \widetilde{z}_{nt})^{\frac{1}{1-\theta}}\, \widetilde{w}_{nt}/\widetilde{z}_{nt} && \text{(C.12)}\\
\text{Hours basket} &: \quad L_t = (n_t \widetilde{z}_{nt})^{\frac{\theta}{\theta-1}}\, \widetilde{\ell}_{nt} && \text{(C.13)}\\
\text{Marginal utility of C} &: \quad \lambda_t = C_t^{-\sigma} && \text{(C.14)}\\
\text{TFP} &: \quad TFP_t = a_t \,(n_t \widetilde{z}_{nt})^{\frac{\theta}{\theta-1}} && \text{(C.15)}\\
\text{GDP} &: \quad GDP_t = W_t L_t && \text{(C.16)}\\
& && \text{(C.17)}
\end{aligned}$$

D Steady State with Balanced Growth Path

We discuss the non-stochastic steady state, its reduced system and the solution. Plugging the free entry and the average dividends of all household members in the steady state, the Euler equation for share holdings becomes:

$$\beta \frac{m_{t+1}}{m_t + m_{et}} E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{W_{t+1} f_{et+1} + \frac{n_{t+1}}{m_{t+1}} \widetilde{d}_{nt+1}}{W_t f_{et}} \right\} = 1. \tag{D.1}$$

Also from the zero cut-off profit condition:

$$\widetilde{w}_{nt}\widetilde{l}_{nt} = \nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} W_t f_{nt}, \tag{D.2}$$

and the average dividends of workers $\widetilde{d}_{nt} = \widetilde{w}_{nt}\widetilde{l}_{nt} - W_t f_{nt}$ in the steady state, we have:

$$\widetilde{d}_{nt} = \left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) W_t f_{nt}.$$

Noting that $\frac{n_t}{m_t} = \left(\frac{\nabla}{\widetilde{z}_{nt}}\right)^{\varepsilon}$ and $W_t = a_t$, the Euler equation for share holdings (D.1) is expressed as:

$$\left[\frac{1+\frac{m_{et}}{m_t}}{\beta g_m}\frac{1}{g_\lambda g_w} - g_{f_e}\right]\left(\frac{\widetilde{z}_{nt+1}}{\nabla}\right)^{\varepsilon} = \left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) g_{fn}\frac{f_{nt}}{f_{et}}.$$

Defining the steady-state growth rate of the average productivity as $\widetilde{z}_{nt+1}/\widetilde{z}_{nt} = g_{\widetilde{z}_n}$ and rewriting the above equation, we get:

$$\widetilde{z}_{nt} g_{\widetilde{z}_n} = \left[\frac{\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) g_{fn}\frac{f_{nt}}{f_{et}}}{\frac{1}{\beta(1-\delta_t)}\frac{1}{g_\lambda g_w} - g_{f_e}}\right]^{\frac{1}{\varepsilon}} \nabla.$$

Since we have with labor supply $\chi_t L_t^{\varphi} = W_t\lambda_t$ so we get $g_\chi g_L^{\varphi} = g_\lambda g_w$, it becomes:

$$\widetilde{z}_{nt} g_{\widetilde{z}_n} = \left[\frac{\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) g_{fn}\frac{f_{nt}}{f_{et}}}{\frac{1+\frac{m_{et}}{m_t}}{\beta g_m}\frac{1}{g_\lambda g_w} - g_{f_e}}\right]^{\frac{1}{\varepsilon}} \nabla.$$

By dividing both sides by $\widetilde{z}_{nt-1} g_{\widetilde{z}_n}$, the growth rate of the average productivity is expressed as:

$$g_{\widetilde{z}_n} = \left[\frac{g_{fn}}{g_{f_e}}\frac{\frac{1+\frac{\frac{m_{et}}{m_t}}{g_{m_e/m}}}{\beta g_m}\frac{1}{g_\chi g_L^{\varphi}} - g_{f_e}}{\frac{1+\frac{m_{et}}{m_t}}{\beta g_m}\frac{1}{g_\chi g_L^{\varphi}} - g_{f_e}}\right]^{\frac{1}{\varepsilon}}. \tag{D.3}$$

Next, with the labor supply equation $\chi_t L_t^{\varphi} = W_t\lambda_t$ and the definition of marginal utility of consumption, $\lambda_t = C_t^{-\sigma}$, we have:

$$C_t = \left(\frac{A_t}{\chi_t L_t^{\varphi}}\right)^{\frac{1}{\sigma}}, \tag{D.4}$$

and its growth rate is:

$$g_C = \left(\frac{g_A}{g_\chi g_L^{\varphi}}\right)^{\frac{1}{\sigma}}.$$

Further, using the definition of wage index $n_t\widetilde{w}_{nt}\widetilde{l}_{nt} = W_t L_t$ and the zero cut-off profit condition (D.2), we have:

$$n_t = \frac{L_t}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} f_{nt}}. \tag{D.5}$$

$$g_n = \frac{g_L}{g_{f_n}}. \tag{D.6}$$

Also using the Pareto distribution, the labor market participation rate is expressed as:

$$\frac{n_t}{m_t} = \left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{-\varepsilon}. \tag{D.7}$$

Thus, we get:

$$g_{m/n} = g_{\widetilde{z}_n}^{\varepsilon}.$$

Furthermore, from the motion $m_{t+1} = (1-\delta_t)\, m_t + (1-\tau_t)\, m_{et}$, we get:

$$\frac{m_{et}}{m_t} = \frac{g_m}{1-\tau_t} - \frac{1-\delta_t}{1-\tau_t}.$$

Thus its growth rate is defined as:

$$g_{m_e/m} = \frac{\frac{g_m}{1-\tau_t} - \frac{1-\delta_t}{1-\tau_t}}{\frac{g_m}{1-\frac{\tau_t}{g_\tau}} - \frac{1-\frac{\delta_t}{g_\delta}}{1-\frac{\tau_t}{g_\tau}}}.$$

Plugging the above expressions, in the labor market clearing condition, $L = \frac{y}{a} + n f_n + m_e f_e$, we have:

$$L_t = \left(\frac{A_t^{\frac{1}{\sigma}-1}}{\chi_t^{\frac{1}{\sigma}}} \frac{1}{1 - \frac{1+\frac{m_{et}}{m_t}\left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{\varepsilon}\frac{f_{et}}{f_{nt}}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}}\right)^{\frac{1}{1+\frac{\varphi}{\sigma}}}.$$

The growth rate of labor supply is therefore:

$$g_L^{1+\frac{\varphi}{\sigma}} = \frac{g_A^{\frac{1}{\sigma}-1}}{g_\chi^{\frac{1}{\sigma}}} \frac{1 - \frac{1+\frac{m_{et}}{m_t}\frac{f_{et}}{f_{nt}}\frac{g_{fn}}{g_{m_e/m}g_{fe}}\left(\frac{\widetilde{z}_{nt}}{\nabla}\frac{1}{g_{\widetilde{z}_n}}\right)^{\varepsilon}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}}{1 - \frac{1+\frac{m_{et}}{n_t}\left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{\varepsilon}\frac{f_{et}}{f_{nt}}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}}. \tag{D.8}$$

Table 3 summarizes the equation at the steady state. There are 26 equations. Endogenously determined variables are $\frac{m_{et}}{m_t}, \frac{n_t}{m_t}$, m_t, L_t, $\widetilde{z}_{nt}$, $\widetilde{l}_{nt}$, n_t, C_t, $\widetilde{d}_{nt}$, W_t, GDP_t, $\frac{GDP_t}{m_t}$, $\frac{GDP_t}{n_t}$ and their growth rates.

Table 3: Balanced-growth steady state

Steady state	Growth rate
$\frac{m_{et}}{m_t} = \frac{g_m}{1-\tau_t} - \frac{1-\delta_t}{1-\tau_t}$	$g_{m_e/m} = \frac{\frac{g_m}{1-\tau_t} - \frac{1-\delta_t}{1-\tau_t}}{\frac{g_m}{1-\frac{\tau_t}{g_\tau}} - \frac{1-\frac{\delta_t}{g_\delta}}{1-\frac{\tau_t}{g_\tau}}}$
$\frac{n_t}{m_t} = \left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{-\varepsilon}$	$g_{n/m} = g_{\widetilde{z}_n}^{-\varepsilon}$
$m_t = n_t \left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{\varepsilon}$	$g_m = g_n g_{\widetilde{z}_n}^{\varepsilon}$
$L_t = \left[\frac{A_t^{\frac{1}{\sigma}-1}}{\chi_t^{\frac{1}{\sigma}}} \frac{1}{1-\frac{1+\frac{m_{et}}{m_t}\left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{\varepsilon}\frac{f_{et}}{f_{nt}}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}} \right]^{\frac{1}{1+\frac{\varphi}{\sigma}}}$	$g_L = \left[\frac{g_A^{\frac{1}{\sigma}-1}}{g_\chi^{\frac{1}{\sigma}}} \frac{1-\frac{1+\frac{m_{et}}{m_t}\frac{f_{et}}{f_{nt}}\frac{g_{fn}}{g_{m_e/m}g_{fe}}\left(\frac{\widetilde{z}_{nt}}{\nabla}\frac{1}{g_{\widetilde{z}_n}}\right)^{\varepsilon}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}}{1-\frac{1+\frac{m_{et}}{n_t}\left(\frac{\widetilde{z}_{nt}}{\nabla}\right)^{\varepsilon}\frac{f_{et}}{f_{nt}}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}} \right]^{\frac{1}{1+\frac{\varphi}{\sigma}}}$
$\widetilde{z}_{nt} = \left[\frac{\left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) g_{fn} \frac{f_{nt}}{f_{et}}}{\frac{1+\frac{m_{et}}{m_t}}{\beta g_m} \frac{1}{g_\chi g_L^{\varphi}} - g_{fe}} \right]^{\frac{1}{\varepsilon}} \frac{\nabla}{g_{\widetilde{z}_n}}$	$g_{\widetilde{z}_n} = \left(\frac{g_{fn}}{g_{fe}} \frac{\frac{1+\frac{\frac{m_{et}}{m_t}}{g_{m_e/m}}}{\beta g_m} \frac{1}{g_\chi g_L^{\varphi}} - g_{fe}}{\frac{1+\frac{m_{et}}{m_t}}{\beta g_m} \frac{1}{g_\chi g_L^{\varphi}} - g_{fe}} \right)^{\frac{1}{\varepsilon}}$
$\widetilde{l}_{nt} = (n_t \widetilde{z}_{nt})^{-\frac{\theta}{\theta-1}} L_t$	$g_{\widetilde{l}_n} = \left(\frac{g_{fn}}{g_{\widetilde{z}_n}}\right)^{\frac{\theta}{\theta-1}} g_L^{1-\frac{\theta}{\theta-1}}$
$n_t = \frac{L_t}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} f_{nt}}$	$g_n = \frac{g_L}{g_{fn}}$
$C_t = \left(\frac{A_t}{\chi_t L_t^{\varphi}}\right)^{\frac{1}{\sigma}}$	$g_C = \left(\frac{g_A}{g_\chi g_L^{\varphi}}\right)^{\frac{1}{\sigma}}$
$\widetilde{d}_{nt} = \left(\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}} - 1\right) W_t f_{nt}$	$g_{\widetilde{d}_n} = g_W g_{f_n}$
$W_t = a_t$	$g_W = g_a$
$GDP_t = W_t L_t$	$g_{GDP} = g_W g_L$
$\frac{GDP_t}{m_t} = \frac{W_t L_t}{m_t}$	$g_{GDP/m} = \frac{g_a g_L}{g_m}$
$\frac{GDP_t}{n_t} = \frac{W_t L_t}{n_t}$	$g_{GDP/n} = \frac{g_a g_L}{g_n}$

E Calibration

Our calibration strategy is as the following. We target the following growth rates $g_{GDP/n}$, $g_{m_e/m}$, $g_{n/m}$ and g_m. These values are function of structural parameters including the growth rates of g_{f_n},g_{f_e}, g_χ, g_δ and g_A. A first group of parameters is calibrated based on the values found in the literature or based on Japanese data or normalized without loss of generality. A second group of parameters namely, g_δ is computed from the theoretical relations: $\frac{m_{et}}{m_t} = \frac{g_m}{1-\delta_t} - 1$ and $g_{m_e/m} = \frac{\frac{g_m}{1-\delta_t}-1}{\frac{g_m}{1-\frac{\delta_t}{g_\delta}}-1}$ given the values of $\frac{m_{et}}{m_t}$, g_m and $g_{m_e/m}$ implied by post-war Japanese data. A third group of parameters, g_A, g_{f_n},g_{f_e}, and χ (which is "adjusted" to imply $L_t = 1$) are estimated to minimize the distance between the weighted average of $g_{GDP/n}$, $g_{m_e/m}$, $g_{n/m}$ and g_m in the data and the implied growth rates predicted by the model. Namely, we minimize the following objective function to estimate the parameters on the four periods:

$$\min_{g_A, g_{f_n}, g_{f_e}, \chi_t} \left(\frac{1}{4} \left(g_{GDP/n} - \bar{g}_{GDP/n} \right) + \frac{1}{4} \left(g_{m_e/m} - \bar{g}_{m_e/m} \right) + \frac{1}{4} \left(g_{n/m} - \bar{g}_{n/m} \right) + \frac{1}{4} \left(g_m - \bar{g}_m \right) \right)$$

F Proof of Proposition 1

By combining the expressions in Table 3, we get $g_m = g_n g_{\tilde{z}_n}^{\varepsilon} = \frac{g_L}{g_{f_n}} g_{\tilde{z}_n}^{\varepsilon}$. Further, this is expressed as:

$$g_m = \frac{\left(\frac{g_A^{\frac{1}{\sigma}-1}}{g_\chi^{\frac{1}{\sigma}}} \frac{1 - \frac{1+\frac{m_{et}}{m_t}\frac{f_{et}}{f_{nt}}\frac{g_{fn}}{g_{m_e/m} g_{fe}}\left(\frac{\tilde{z}_{nt}}{\nabla}\frac{1}{g_{\tilde{z}_n}}\right)^{\varepsilon}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}}{1 - \frac{1+\frac{m_{et}}{n_t}\frac{f_{et}}{f_{nt}}\left(\frac{\tilde{z}_{nt}}{\nabla}\right)^{\varepsilon}}{\nabla^{\frac{\theta(1+\varphi)}{1+\theta\varphi}}}} \right)^{\frac{1}{1+\frac{\varphi}{\sigma}}}}{g_{f_n}} g_{\tilde{z}_n}^{\varepsilon}.$$

With $g_\delta = g_\tau = 1$, we have $g_{m_e/m} = 1$. Note also that:

$$g_{\tilde{z}_n} = \left[\frac{g_{fn}}{g_{f_e}} \frac{\frac{1+\frac{\frac{m_{et}}{m_t}}{g_{m_e/m}}}{\beta g_m} \frac{1}{g_\chi g_L^{\varphi}} - g_{f_e}}{\frac{1+\frac{m_{et}}{m_t}}{\beta g_m} \frac{1}{g_\chi g_L^{\varphi}} - g_{f_e}} \right]^{\frac{1}{\varepsilon}}.$$

So with $g_\delta = g_\tau = 1$, we have $g_{\tilde{z}_n} = \left(\frac{g_{fn}}{g_{fe}}\right)^{\frac{1}{\varepsilon}}$. This implies:

$$g_m = \frac{\left[\frac{g_a^{\frac{1}{\sigma}-1}}{g_\chi^{\frac{1}{\sigma}}} \right]^{\frac{1}{1+\frac{\varphi}{\sigma}}}}{g_{f_e}}.$$

So the condition of population growth such that $g_m > 1$ is:

$$g_{f_e} g_a^{\frac{\sigma-1}{\sigma+\varphi}} g_\chi^{\frac{1}{\sigma+\varphi}} < 1$$

Also GDP per capita is expressed as:

$$g_{GDP/m} = \frac{g_a g_{f_n}}{\left(\frac{g_{fn}}{g_{fe}} \frac{\frac{1+\frac{\frac{m_{et}}{m_t}}{g_{m_e/m}}}{\beta g_m} \frac{1}{g_\chi g_L^\varphi} - g_{fe}}{\frac{1+\frac{m_{et}}{m_t}}{\beta g_m} \frac{1}{g_\chi g_L^\varphi} - g_{fe}} \right)}.$$

With $g_\delta = g_\tau = 1$, we have $g_{m_e/m} = 1$. The condition at which GDP per capita grows $(g_{GDP/m} > 1)$ is given by:

$$g_a g_{f_e} > 1.$$

G Proof of Proposition 2

By combining the expressions in Table 3, the labor market participation rate is:

$$g_{n/m} = g_{\tilde{z}_n}^{-\varepsilon}$$

With $g_\delta = g_\tau = 1$, we have $g_{m_e/m} = 1$ and thus $g_{\tilde{z}_n} = \left(\frac{g_{fn}}{g_{fe}}\right)^{\frac{1}{\varepsilon}}$. Thus, the above expression is:

$$g_{n/m} = \frac{g_{f_e}}{g_{fn}}.$$

So the condition under which the labor market participation rate increases over time $(g_{n/m} > 1)$ is:

$$g_{f_e} > g_{fn}.$$

Also GDP per worker is:

$$g_{GDP/n} = \frac{g_a g_L}{g_n} = \frac{g_a g_L}{\frac{g_L}{g_{fn}}} = g_a g_{f_n}.$$

So the condition under which GDP per worker grows is:

$$g_a g_{f_n} > 1.$$

The above condition holds independently the value of g_δ.

H Income distribution

We characterize the relation between wage income distribution and the distribution of the idiosyncratic productivity level z. The FOC with respect to $w_t(j,z)$ is given by:

$$\frac{\eta\theta}{\theta-1} z l_t(j,z)^{-\frac{1}{\theta}} H_t(j)^{\varphi+\frac{1}{\theta}} = \lambda_t(j) w_t(j,z).$$

Plugging the labor demand $l_t(j,z) = \left(\frac{w_t(j,z)}{W_t(j)z}\right)^{-\theta} L_t(j)$, we get:

$$w_t(j,z) l_t(j,z) = w_t(j,z)^{1-\theta} z^{\theta} \left[\frac{\eta\theta}{\theta-1} W_t^{\theta-1}(j) \frac{L_t(j)^{\varphi+\frac{1}{\theta}}}{\lambda_t(j)}\right].$$

Further we have:

$$\frac{w_t(j,z)}{w_t(\tilde{z}_t)} = \left(\frac{z}{\tilde{z}_t}\right)^{\frac{\theta\varphi}{1+\theta\varphi}}.$$

Thus,

$$w_t(j,z) l_t(j,z) = z^{\left(\frac{\varphi(1-\theta)}{1+\theta\varphi}+1\right)\theta} \left[\frac{\eta\theta}{\theta-1} \tilde{z}_t^{-\frac{\theta\varphi(1-\theta)}{1+\theta\varphi}} \left(\frac{w_t(\tilde{z}_t)}{W_t(j)}\right)^{1-\theta} \frac{L_t(j)^{\varphi+\frac{1}{\theta}}}{\lambda_t(j)}\right].$$

The term in the square bracket is independent of z. In equilibrium, households are symmetric and we thus drop the index j. Further, we know that the log of z has the standard deviation of $1/\varepsilon$ since z follows a Pareto distribution with the shape coefficient ε. Finally, the standard deviation of the log of wage income is given by:

$$\text{Std}[\log w_t(z) l_t(z)] = \left(\frac{\varphi+1}{1+\theta\varphi}\right)\frac{\theta}{\varepsilon}.$$

For the purpose of calibration, we back out the value of ε as:

$$\varepsilon = \left(\frac{\varphi+1}{1+\theta\varphi}\right)\frac{\theta}{\text{Std}[\log w_t(z) l_t(z)]}.$$

For the empirical standard deviation, we use the National Survey of Family Income and Expenditure (NSFIE) data from Japan, as provided by Kitao and Yamada (2019). Specifically, we calculate the average standard deviation of log incomes for the years 1984, 1989, 1994, 1999, 2004, 2009, and 2014. We prefer using the income distribution, which potentially includes sources of income beyond just wage income, over the earnings distribution due to greater data coverage. The income distribution covers the entire range from the botttom to the top 1%, whereas the earnings distribution is only available for the top 40 % and higher.