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Strategy Assortativity and the Evolution of Parochialism

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Strategy Assortativity and the Evolution of Parochialism

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Abstract

This paper investigates the role of strategy assortativity for the evolution of parochialism. Individuals belonging to different groups are matched in pairs to play a prisoner dilemma, conditioning their choice on the identity of the partner. Strategy assortativity implies that a player is more likely to be matched with someone playing the same strategy. We find that, if the degree of strategy assortativity is sufficiently high, then parochialism (i.e., cooperate with your own group and defect with others) spreads over a group, while egoism (i.e., defect with everyone) emerges otherwise. Notably, parochialism is more likely to emerge in a smaller group.

JEL classification codes: C72; C73; Z10.

Keywords: prisoner dilemma; cooperation; in-group favoritism; cultures; asymptotic stability.

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1 Introduction

Assortative matching refers to the fact that individuals are more likely to be matched with people similar to them in some relevant respect. This property of matching processes has been a subject of interest in connection with the evolution of prosocial behavior.¹

In many analyses explicitly considering assortative matching, similarities refer to belonging to the same cultural group, the same social or ethnic group, or the same religion (McPherson et al., 2001). However, assortative matching can be driven by similar interests or preferences, irrespective of the cultural, ethnic or social group of the individuals. In this latter case, the concept of assortativity refers to having the same behavior, even across different social groups.

Examples might be drawn where assortative matching on preferences takes place across different groups. Probably the most famous is the Christmas Truce during the First World War: in the week before the 25th of December 1914, all over the Western Front, many French, German and British soldiers with strong preferences for celebrating Christmas crossed trenches to exchange greetings, foods, and play football. More recently, in the Kaduna state of Nigeria, where the religious conflict is harsh, Muslims and Christians often meet together during the church service, revealing a strong preference for having a good relationship among all the faithful. These examples are, evidently, exceptional cases. They are not aimed at convincing that assortativity does not typically occur between the same cultural, social or ethnic group. They should only provide evidence that is not structurally so: there are cases where assortativity is driven by behaviors or preferences, and the assortativity on groups is at most a result. This is exactly what comes out from our model, where type assortativity endogenously emerges in equilibrium.

Also, assortativity by groups excludes a few extreme cases, albeit socially relevant, where

¹For a detailed review of assortativity in evolutionary game theoretical models see Section 3 of [Newton \(2018\)](#).

some individuals may dislike or devalue their own cultural group. This is the case of “cultural cringe” ([Phillips, 2006](#)): the desire to get along with individuals belonging to different cultures, a common phenomenon now in, e.g., Australia, Canada and Brazil. Assuming assortativity by groups makes it hard to account for these social phenomena.

In this paper we investigate the evolution of cooperation considering assortative matching driven by the behavior code, to which we refer as “strategy assortativity”. Along this concept, individuals are more likely to be matched together if their behavior code is similar, regardless of their social group.

Specifically, we consider a setting where individuals belong to one of two social groups and are drawn to play a prisoner dilemma in pairs. With two social groups, there are four possible strategies for a generic member of each social group: cooperation with everyone (cooperation), cooperation only with the own group (parochialism), cooperation only with the other group (anti-parochialism) and defection with everyone (egoism).

In the context where individuals hang out into different places, strategy assortativity may arise because preferences determine where they gather. An example might be handy to get the idea. Consider a population of Christians and Muslims who have heterogeneous preferences about with whom to cooperate. People who want to cooperate with Christians but not with Muslims tend to go to a church, people who want to cooperate with Muslims but not with Christians tend to go to a mosque, people who want to cooperate with everybody tend to go to non-religious volunteering associations, and people who do not want to cooperate with anyone tend to go to bars, clubs or the like. Importantly, people going in the same place are more likely to interact.

We find that, while equilibria of cooperation or anti-parochialism are always selected against – no matter how strong is assortativity – an egoist equilibrium might be eroded over time by parochialism. This is consistent with recent experimental evidence, showing that parochialism is ubiquitous in 42 nations ([Romano et al., 2021](#)).

In fact, when strategy assortativity is strong enough, parochialism is always favored by evolution. These results may be explained as follows. In a cooperative equilibrium, mutant parochialists of the same group have an advantage with respect to cooperators of any group since, in assortative matchings, they receive all the benefits of cooperation as cooperators do while, in random matchings, they save on the costs of cooperation when interacting with cooperators belonging to other groups. For the same argument, the advantage is even greater in an anti-parochial equilibrium. In an egoist equilibrium, mutant parochialists of the same group have an advantage with respect to egoists of any group, if strategy assortativity – and so the chance that a mutant meets another mutant – is strong enough. Indeed, in case of assortative matching, the mutants cooperate with each other obtaining a greater benefit than egoists do, which might be greater than the cost of cooperating in case of random matching with a member of the mutants' group. These results have an interesting consequence: when strategy assortativity is strong enough to induce the emergence of parochialism, it also leads *de facto* to type assortativity.

We then investigate the role played by group size. We find that it is more likely that parochialism emerges as a globally asymptotically stable state in a small group. This can be explained with the fact that parochialism is more costly for individuals belonging to a larger group. On the one hand, the costs and benefits are independent of group size in case of assortative matching. On the other hand, the benefits do not depend on own group size – they only depend on the fraction of individuals who cooperate with members of such group. Therefore, the probability to pay the cost of cooperation is larger for the parochial members of larger groups.

Next, we focus on the analysis of average cooperation in the entire population. In this case, with two groups, the increase of relative size of one population implies the decrease in relative size of the other. We find that average cooperation is larger for higher levels of strategy assortativity. The impact of the relative population size of the two groups is more

articulated. In case where strategy assortativity is sufficiently high, and parochialism in all groups is a globally asymptotically stable strategy, average cooperation is larger for greater differences in size between the two groups. If the level of strategy assortativity is such that the globally asymptotically stable strategy is parochialism in one group and egoism in the other, consistently with the results above, then the parochial group is the minority. Hence, average cooperation increases with the size of the minority group, as this is the group that pushes cooperation.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the relevant literature and the connection with this work. Section 3 describes the framework, while Section 4 shows the baseline results. Section 5 focuses on the analysis of cooperation. Section 6 generalizes the results to the case of a population composed by more than two groups. Section 7 concludes by discussing our results and sketching future research lines.

2 Literature

The present paper contributes to the literature on parochialism, which is the tendency to collaborate with individuals of the same group and not with others. Parochialism has been analyzed from different perspectives. One strand analyzes parochialism in the context of between-group cooperation (see Dyble, 2021, Garcia and van den Bergh, 2011, Choi and Bowles, 2007 and Choi and Bowles, 2003, among others). A different approach focuses on cooperation in frameworks where the strategy depends on spatial interaction and geographical proximity between players (see Berg et al., 2021, Bowles and Gintis, 2004a, Bowles and Gintis, 2004b, McElreath et al., 2003 and Eshel et al., 1998, among others). In the present paper, we abstract away from both group selection and distance: parochialism emerges as a social outcome, given a sufficiently high degree of strategy assortativity.

This paper also contributes to the stream of literature on strategy assortativity. The seminal contribution in this strand is Bergstrom (2003), who introduces an index of as-

sortativity in a prisoner dilemma, which is given by the difference between the probability that a cooperator meets a cooperator and the probability that a defector meets a cooperator. In a later contribution, [Bergstrom \(2013\)](#) discusses assortativity in relation to different matching processes: first, the two-pool assortative matching, where an individual matches with some probability from an “assortative pool” consisting only of one’s own type, and with the complementary probability from a “random pool”; second, the strangers-in-the-night matching, where individuals meet randomly, and they accept the match with similar partners with higher probability. Among recent contributions on strategy assortativity, [Nax and Rigos \(2016\)](#) and [Wu \(2016\)](#) analyze the evolution of endogenous action assortativity through democratic consensus in social dilemmas and coordination games, respectively. [Xu et al. \(2019\)](#) analyze strategy assortativity among investors in financial markets. A general approach to assortativity is put forward by [Van Veelen \(2011\)](#), which generalizes the findings in [Van Veelen \(2009\)](#) regarding the validity of Hamilton’s rule ([Hamilton, 1964](#)), allowing for matching among more than two individuals.

The starting point of the present analysis is [Bilancini et al. \(2018\)](#). They assume that interacting among members of different cultures is costly, because of culture-related norms or habits. To do so, they consider two exogenous cultures, where individuals face a cost of cultural intolerance if they have to interact with someone belonging to the different culture. Unlike [Bilancini et al. \(2018\)](#), we abstract away from cultural intolerance, whose presence would in fact reinforce our findings but limit their generality. Rather, we argue that cultural intolerance may arise as an outcome. In addition, we focus on “strategy” rather than “action” assortativity, that is, a more nuanced concept that allows assortativity from individuals of different cultures but similar behavior. Finally, our analysis covers both cases of two and more than two groups.

The analysis of assortative matching based on group affiliation or cultural traits has developed in at least two important directions: individuals may match assortatively because

they are willing to do so, or because they interact with neighbors and neighbors are similar to them. The first reason points to homophily, i.e., *love of the same*, which is a well-documented phenomenon in social life (Currarini et al., 2009). The second reason points to family ties, in that individuals tend to interact often with relatives, and relatives are more likely to have one’s own type (Bergstrom, 1995, Alger and Weibull, 2010, Lehmann et al., 2015, Alger et al., 2020).

Alger and Weibull (2013, 2016) deserve a specific discussion. In their framework, assortativity is preference based: different individuals may have different preferences, and individuals having the same preferences are more likely to interact together. The domain of preferences encompasses the actions to be played: hence, individuals may exhibit different “moral preferences”, yielding a different willingness to cooperate. In this context, they find that, under incomplete information, moral preferences allow cooperation to spread in the population and to resist to invasions. Newton (2017) embeds the evolution of assortativity into Alger and Weibull (2013)’s model, showing that their results do not extend to this setting. In our paper we assume different cultural groups, but we consider assortativity taking place only on strategies. While we do not explicitly consider preferences over strategies, strategy adoption may be explained as the result of preferences. Differently from Alger and Weibull (2013), we focus on the evolution of strategies only, keeping types fixed. We stress that types are public information, implying that behaviors are conditional on type. In this setting, we endogenously obtain the evolution of type assortativity and parochialism.

Finally, our paper may be related to the literature on the cultural transmission of values. Relevant examples are Cavalli-Sforza and Feldman (1981), Boyd and Richerson (1988), and Bisin and Verdier (2001) (see Cheung and Wu, 2018 for a continuous-trait extension of the binary-trait model). More recently, Wu and Zhang (2021) have shown the crucial role played by assortativity on the dynamics of cultural transmission. Our model differs from those in this stream of literature in that, if we interpret types as cultural groups (as done in Bilancini

et al., 2018), then culture is exogenously given (possibly, due to a relatively short time horizon of the analysis), while only behaviors are allowed to evolve over time.

3 The model

3.1 Population

We study a large population, with mass normalized to 1, composed by individuals belonging to one of two cultural groups, i or j , which are public information. We denote by g a generic group, with $g \in \{i, j\}$. Without loss of generality, we assume that group i is larger than group j . In particular, a share of population β^i belongs to group i , while the share $\beta^j = 1 - \beta^i$ belongs to group j , with $\beta^i > \beta^j$.

3.2 Stage game

Individuals are matched in pairs to play a prisoner dilemma with additive payoffs: in each match, the possible actions are “cooperate” (C) or “defect” (D), as illustrated in the following payoff matrix:

	C	D
C	$b - c, b - c$	$-c, b$
D	$b, -c$	$0, 0$

3.3 Strategies

Individuals can condition the action played in the stage game to the partner being a member of group i or j . In particular, an individual follows a strategy $x \in \{CC, CD, DC, DD\}$ where

the first action is the one played against an individual from group i and the second action is the one played against an individual of group j . The fraction of individuals belonging to group i (j is analogous) who adopt each strategy are:

- s_{CC}^i : i type that cooperates with i and j (*cooperation*);
- s_{CD}^i : i type that cooperates with i and does not cooperate with j (*parochialism*);
- s_{DC}^i : i type that cooperates with j for every $j \in n$, and does not cooperate with i (*anti-parochialism*);
- s_{DD}^i : i type that does not cooperate with anyone (*egoism*),

with

$$s_{CC}^i + s_{CD}^i + s_{DC}^i + s_{DD}^i = 1.$$

A state is denoted by the pair $s = (s^i, s^j)$ where $s^i = (s_{DD}^i, s_{CC}^i, s_{CD}^i, s_{DC}^i)$ and $s^j = (s_{DD}^j, s_{CC}^j, s_{CD}^j, s_{DC}^j)$. The set of all possible states is denoted by S .

We denote the fraction of individuals from group i who play strategy x in state $s \in S$ as:

$$\eta_{i|x}(s) = \frac{\beta^i s_x^i}{\beta^i s_x^i + \beta^j s_x^j},$$

which embeds all the possible cases. The fraction of individuals from group j who play strategy x in state s , $\eta_{j|x}(s)$, is computed analogously.

3.4 Matching process

The main feature of this model is the adoption of the following concept to strategy assortativity: individuals are more likely to be matched together if they follow the same strategy x .

The random matching follows the two-pool assortative matching process with uniform assortativity ([Cavalli-Sforza and Feldman, 1981](#)): with probability $p \in (0, 1)$ an individual

is matched with someone who adopts the same strategy x (i.e., drawn from an assortative pool), while with probability $1 - p$ he is matched with a random partner (i.e., drawn from the pool of all the agents who do not match assortatively).

3.5 Expected payoffs

We denote the expected payoff of an agent adopting strategy x in state s and belonging to group i (j is analogous) with $\pi_x^i(s)$. Explicit formulas for the expected payoffs of an individual of group i who adopts strategy CC , CD , DC and DD are, respectively:

$$\begin{aligned}\pi_{CC}^i(s) &= pb - c + \\ &\quad (1 - p) [\beta^i (s_{CC}^i + s_{CD}^i) + \beta^j (s_{CC}^j + s_{CD}^j)] b,\end{aligned}$$

$$\begin{aligned}\pi_{CD}^i(s) &= p(b - \eta_{i|CDC}) + \\ &\quad (1 - p) [(\beta^i (s_{CC}^i + s_{CD}^i) + \beta^j (s_{CC}^j + s_{CD}^j)) b - \beta^i c],\end{aligned}$$

$$\begin{aligned}\pi_{DC}^i(s) &= (1 - p) [(\beta^i (s_{CC}^i + s_{CD}^i) + \beta^j (s_{CC}^j + s_{CD}^j)) b - \beta^j c] \\ &\quad - p(1 - \eta_{i|DC}) c,\end{aligned}$$

$$\pi_{DD}^i(s) = (1 - p) [\beta^i (s_{CC}^i + s_{CD}^i) + \beta^j (s_{CC}^j + s_{CD}^j)] b.$$

The details regarding the derivation of the above formulas may be found in [Appendix A](#).

3.6 Asymptotic stability

We focus on dynamics where strategies evolve over time satisfying payoff monotonicity ([Weibull, 1995](#)), keeping group sizes are fixed. This is consistent with [Bilancini et al. \(2018\)](#),

where the horizon is sufficiently long such that selection operates on strategies but not so long that it also operates on cultural types. Time is continuous and denoted with t , with $s(t)$ indicating the state of the system at time t .

In a payoff monotonic dynamic, a pure strategy with a higher payoff always has a higher growth-rate than a pure strategy with a lower payoff. In the following definition, \dot{s}_x^g denotes the time derivative of the fraction of agents in group g playing strategy x .

DEFINITION 1 (Payoff monotonicity). *A dynamic is payoff monotone if for $g \in \{i, j\}$ and $x \in \{CC, CD, DC, DD\}$, $\dot{s}_x^g/s_x^g > \dot{s}_{x'}^g/s_{x'}^g$ if and only if $\pi_x^g(s) > \pi_{x'}^g(s)$.*

In the remaining of the paper, we will rely on global asymptotic stability for stating our results (see [Sandholm, 2010](#), for a thorough review of stability concepts in dynamic systems). A state is globally asymptotically stable if any trajectory in the interior of the state space eventually converges to the same equilibrium point. We denote by \hat{S} the interior of the state space, i.e., $\hat{S} = \{s \in S : s_x^g > 0 \text{ for any } g \text{ and } x\}$.

DEFINITION 2 (Global asymptotic stability). *A state s^* is globally asymptotically stable if, for any $s \in \hat{S}$, we have $s(t) \rightarrow s^*$ as $t \rightarrow \infty$.*

4 Results

The following lemma compares the expected payoffs of the different strategies.

LEMMA 1. *For every $s \in \text{int}(S)$, $g \in \{i, j\}$:*

- (i) $\pi_{CD}^g(s) > \pi_{CC}^g(s)$;
- (ii) $\pi_{CD}^g(s) > \pi_{DC}^g(s)$;
- (iii) $\pi_{DC}^g(s) \leq \pi_{DD}^g(s)$ if and only if $p \leq \hat{p}^g$, where

$$\hat{p}^g \equiv \frac{c\beta^g}{b - (\beta^g - 1)c}.$$

Proof. In Appendix B. □

Lemma 1 shows that, while cooperation and anti-parochialism are always dominated by parochialism, the latter always outperforms egoism if the degree of strategy assortativity is above a certain threshold, while the opposite holds if the degree of strategy assortativity is below the threshold. Notice that \hat{p}^g differs in the two groups, in that it depends on the relative size of the group. Indeed, a close inspection shows that $\hat{p}^i > \hat{p}^j$ because $\beta^i > \beta^j$, as formally stated in the following corollary.

COROLLARY 1. *The larger the size of group g , the larger the threshold \hat{p}^g .*

Proof. In Appendix B. □

We are now able to derive which states are globally asymptotically stable.

PROPOSITION 1. *The globally asymptotically stable state s is such that*

- $s_{CD}^i = 1$ and $s_{CD}^j = 1$, if $p > \hat{p}^i$;
- $s_{DD}^i = 1$ and $s_{CD}^j = 1$, if $\hat{p}^j < p < \hat{p}^i$;
- $s_{DD}^i = 1$ and $s_{DD}^j = 1$, if $p < \hat{p}^j$.

Proposition 1 shows that parochialism is globally asymptotically stable in a group if the degree of strategy assortativity is strong enough. The intuition is simple: with a higher strategy assortativity, parochialism yields a higher expected payoff because a higher p indirectly amounts to having a higher chance of meeting a partner of the same type, who will cooperate. The effect of an increase in the degree of strategy assortativity on the payoff of defection is, if any, negative, in that it implies a higher chance of meeting a partner of the same type, who will defect.

Also, Proposition 1 reveals that the outcome in case of an intermediate degree of strategy assortativity is such that parochialism emerges in the smallest group, and defection in the

largest group. The reason why parochialism is more likely to emerge in the smallest group is that the cost of parochialism is lower the smaller the group size. This is due to the fact that, when matching occurs randomly, the probability to meet an individual of own type is given by the relative group size. This, in turn, determines the frequency, and hence the cost, of cooperation in case of parochialism.

5 Analysis of cooperation

Proposition 1 shows that evolution favors the emergence of cooperation among the members of a certain group g provided that the level of strategy assortativity p is sufficiently high, i.e., greater than the threshold \hat{p}^g . Importantly, cooperation is not unconditional towards everybody, but it only comes in the form of parochialism, meaning that the members of g only cooperate with other members of g . Hence, for $p > \hat{p}^g$, average cooperation for members of g is:

$$\bar{C}^g = p + \beta^g(1 - p). \quad (1)$$

Intuitively, with probability p an individual from g is matched with another individual from g , and both cooperate. With probability $(1 - p)$ instead, the matching is random and, hence, cooperation occurs only if another member of g is met, which occurs with probability β^g . Average cooperation of individuals in g changes according to variations of group size and the degree of strategy assortativity. Differentiating (1) with respect to both β^g and p we get, for $p > \hat{p}^g$:

$$\frac{\partial \bar{C}^g}{\partial \beta^g} = (1 - p) > 0; \quad (2)$$

$$\frac{\partial \bar{C}^g}{\partial p} = 1 - \beta^g > 0. \quad (3)$$

An increase of both the size of the group and the degree of strategy assortativity increase average cooperation of group g , as it raises the chance of being matched with one member of group g , but for different reasons. By (2), a larger group g implies a higher probability to be

randomly matched with one belonging to g . By (3), a higher degree of strategy assortativity increases the probability that a matching is assortative in strategy rather than random and hence, by the nature of the equilibrium from Proposition 1, also assortative in type.

From (2), average cooperation in group g increases with the size of the group, since the probability of being randomly matched with another member of g increases. From (3), average cooperation in group g increases with the degree of strategy assortativity as well, since the probability of being matched with another member of g increases as the degree of strategy assortativity increases.

However, for $p < \hat{p}^g$, we have $\bar{C}^g = 0$ and $\partial \bar{C}^i / \partial \beta^g = \partial \bar{C}^g / \partial p = 0$. When strategy assortativity is low no cooperation emerges and, hence, little differences in group size or strategy assortativity are not going to affect group cooperation.

The next step is to look at average cooperation of the whole population. With two groups, i and j , average cooperation of the whole population depends on the level of both \hat{p}^i and \hat{p}^j . Three qualitatively different cases are possible. In the first, $p > \hat{p}^i > \hat{p}^j$ so that members of both groups cooperate when they meet another member of their own group. In the second case, $\hat{p}^i > p > \hat{p}^j$ so that only members of group j , the smaller group, cooperate among themselves. In the last case, $\hat{p}^i > \hat{p}^j > p$ so that nobody cooperates. Note that $\beta^j = 1 - \beta^i$, namely a larger size of one group necessarily implies a smaller size of the other. For $p > \hat{p}^i > \hat{p}^j$ average cooperation in the population is given by:

$$\bar{C} = \beta^i [p + \beta^i(1 - p)] + (1 - \beta^i) [p + (1 - \beta^i)(1 - p)], \quad (4)$$

which, differentiating with respect to β^i , gives:

$$\frac{\partial \bar{C}}{\partial \beta^i} = 2(1 - p)(2\beta^i - 1) > 0. \quad (5)$$

Intuitively, a greater size of the larger group leads to more cooperation on average because the random pairs are more likely to be formed by members of the same group, who hence

cooperate with each other. By contrast, differentiating (4) with respect to p yields:

$$\frac{\partial \bar{C}}{\partial p} = 2(1 - \beta^i) \beta^i > 0, \quad (6)$$

in line with effect of p on the average cooperation for each single group. These findings are summarized by the following Proposition 2.

PROPOSITION 2. *For $p > \hat{p}^i > \hat{p}^j$, average cooperation is larger the larger is the difference in size between the two groups $\beta^i - \beta^j$. In addition, average cooperation is larger the larger the level of strategy assortativity p .*

Consider next the case where $\hat{p}^i > p > \hat{p}^j$. In this case, average cooperation amounts to

$$\bar{C} = \bar{C}^j = (1 - \beta^i) [p + (1 - \beta^i)(1 - p)]. \quad (7)$$

Differentiating with respect to β^i we get:

$$\frac{\partial \bar{C}}{\partial \beta^i} = \frac{\partial \bar{C}^j}{\partial \beta^i} = 2\beta^i - 2\beta^i p + p - 2 < 0.$$

Since cooperation only occur by members of group j , an increase in the proportion of group i , where no cooperation at all occurs, necessarily decreases average cooperation. Moreover, differentiating with respect to p we get:

$$\frac{\partial \bar{C}}{\partial p} = \frac{\partial \bar{C}^j}{\partial p} = (1 - \beta^i) \beta^i > 0.$$

Average cooperation increases with p since a greater p increases the frequency of interactions between members of group j who only cooperate among themselves. These findings are summarized by the following Proposition 3.

PROPOSITION 3. *For $\hat{p}^i > p > \hat{p}^j$, average cooperation is smaller the larger is the difference in size between the two groups $\beta^i - \beta^j$. In addition, average cooperation is larger the larger the level of strategy assortativity p .*

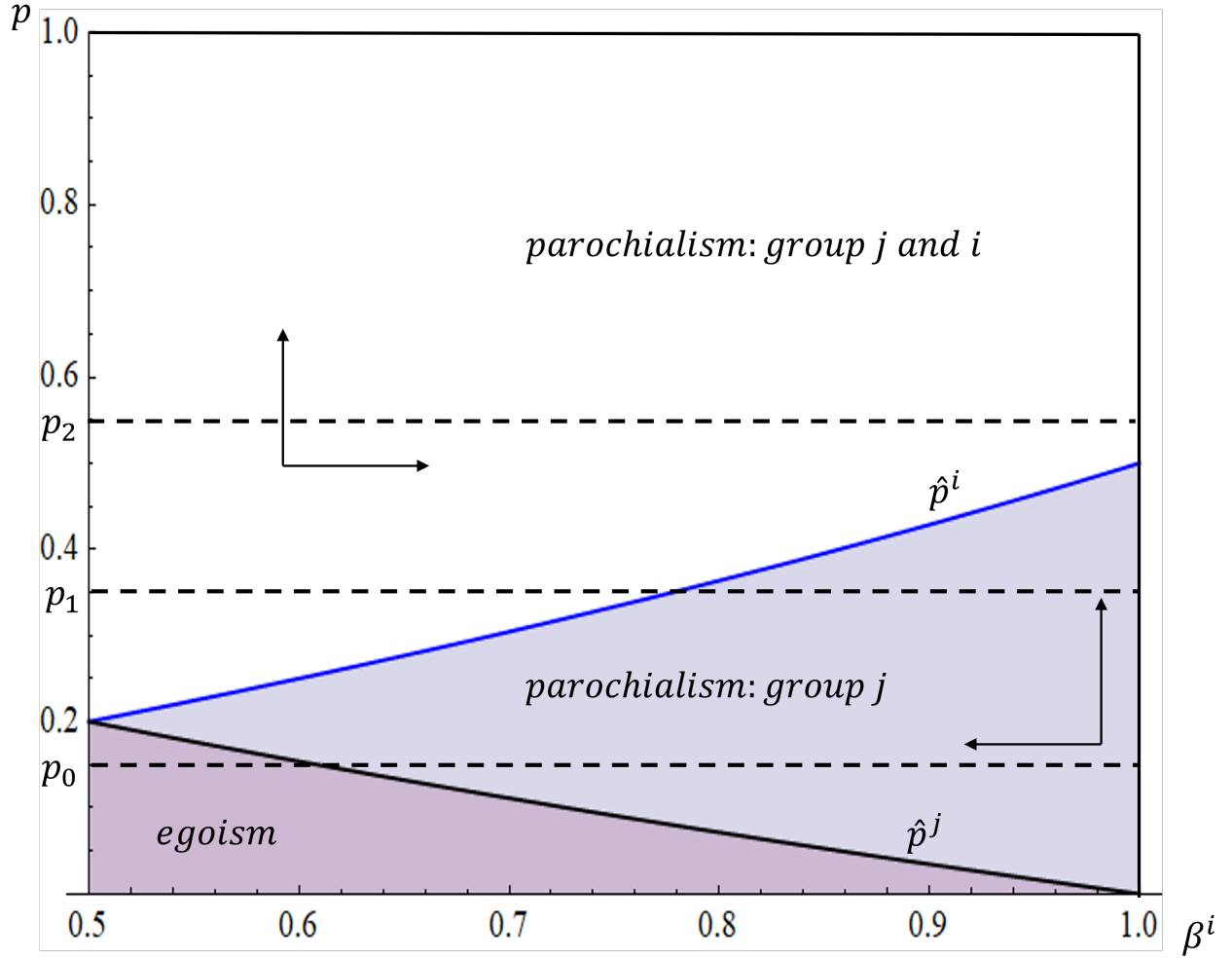


Figure 1: Parochialism in the two groups as a function of the degree of strategy assortativity p and the size of the majority group β^i , for $b = 2$ and $c = 1$.

The effect of a change in population size of group i on cooperation depends on whether the other group j is cooperating or not. If group i is not cooperating, the increase in the size of group j monotonically increases the average cooperation in the whole population. If also group i is cooperating then, due to parochialism, average cooperation depends on the frequency of interactions among members of the same group. In this case greater average cooperation can be attained increasing the difference in the size between the two groups.

Fig. 1 summarizes these results, with arrows indicating the increase in average cooperation.

For a level of strategy assortativity p above the maximum of \hat{p}^i (see p_2 in Fig. 1), average cooperation reaches its maximum when $\beta^i = 1$: for such a high level of strategy assortativity, the majority groups is always parochial, even when its relative size tends to 1 (see the dotted line in Fig. 2).

For any level of p in between the maximum of \hat{p}^i and the maximum of \hat{p}^j (see p_1 in Fig. 1), the level of β that reaches maximum cooperation is $\beta(\hat{p}^i)^{-1}$, when both groups are parochial (see the dashed line in Fig. 2).

Finally, for any level of p lower than the maximum of \hat{p}^j (see p_0 in Fig. 1), the level of β that reaches maximum cooperation is $\beta(\hat{p}^j)^{-1}$, when group j is parochial (see the solid line in Fig. 2).

6 Cosmopolitan society

In this section we extend the analysis by considering n groups instead of two. We show that the results are consistent with the baseline analysis, albeit it is harder to elicit insights in the analysis of cooperation.

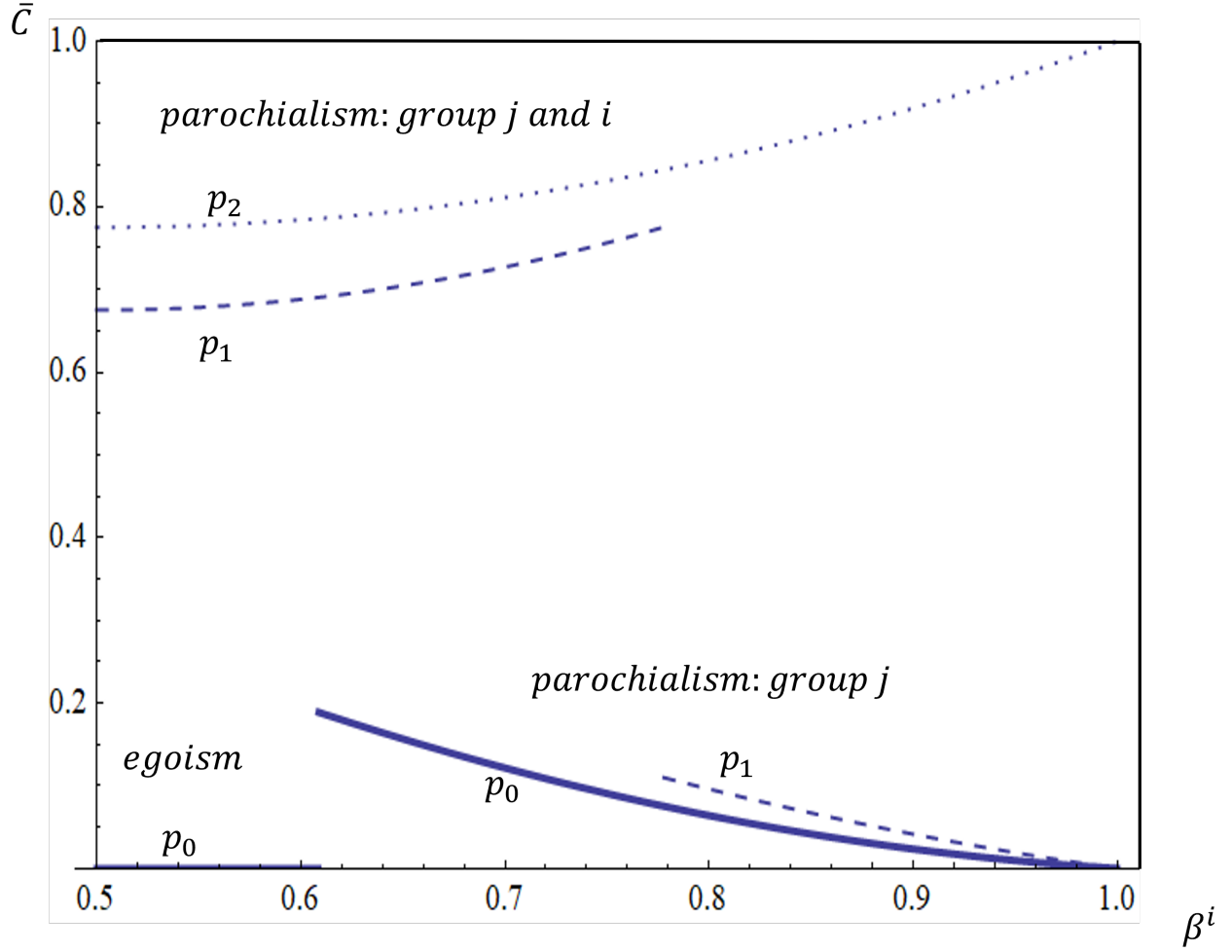


Figure 2: Average cooperation when $b = 2$ and $c = 1$ for three values of p : $p_1 = 0.15$, $p_2 = 0.35$, $p_3 = 0.55$.

6.1 Population, game and matching

In a cosmopolitan society, the population mass is still normalized to one, while the size of a generic group g is denoted as β^g , with $\sum_{g=1}^n \beta^g = 1$. Individuals are still matched in pair to play the same prisoner dilemma. The matching process is analogous to the one adopted in the baseline case.

6.2 Strategies

With n groups, of course, the possible combinations of strategies will be too many to be fully investigated. We will thus focus on the case in which the only possible strategies, for each member of any group $g = 1, \dots, n$, are :

- cooperates with anyone (*cooperation*), whose fraction is s_{CC}^g ;
- cooperates with g and defect with anyone else (*parochialism*), , whose fraction is s_{CD}^g ;
- cooperates with anyone but g , (*anti-parochialism*), whose fraction is s_{DC}^g ;
- defect with anyone (*egoism*), whose fraction is s_{DD}^g ,

and where

$$s_{CC}^g + s_{CD}^g + s_{DC}^g + s_{DD}^g = 1.$$

In turn, the fraction of individuals from group i in states x are:

$$\eta_{i|x} = \frac{\beta^i s_x^i}{\sum_{g=1}^n \beta^g s_x^g}.$$

6.3 Expected payoffs

Having outlined the assumption about strategies, in a setting with n groups, the expected payoffs of an individual of group i who adopts strategy cooperation, parochialism, anti-parochialism and egosim are, respectively (see Appendix A for details),

$$\begin{aligned}
\pi_{CC}^i(s) &= pb + (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - c, \\
\pi_{CD}^i(s) &= p(b - \eta_{i|CD}c) + (1-p) \left[\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \beta^i c \right], \\
\pi_{DC}^i(s) &= (1-p) \left(\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \sum_{j \neq i} \beta^j c \right) - p(1 - \eta_{i|DC})c, \\
\pi_{DD}^i(s) &= (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b.
\end{aligned}$$

6.4 Evolution and selection

In this section we analyze the equilibrium results in a cosmopolitan society. The relationships between expected payoffs do not change compared to Lemma 1, see Appendix B for details. By contrast, to find the global asymptotic stability in an n -group population, it might be of help to sort the groups based on their relative size.

ASSUMPTION 1. *Let $n > 2, n \in \mathbb{N}$ and $\beta^n > \beta^{n-1} > \dots > \beta^1$.*

With Assumption 1, we may derive the equivalent of Proposition 1 for a cosmopolitan society.

PROPOSITION 4. *Suppose Assumption 1 holds. The globally asymptotically stable state s is such that*

- $s_{CD}^g = 1$ for every g , if $p > \hat{p}^n$;
- $s_{DD}^g = 1$ for every $g \geq i$ and $s_{CD}^{g'} = 1$, for every $g' \leq i$, if $\hat{p}^i < p < \hat{p}^{i+1}$;
- $s_{DD}^g = 1$ for every g , if $p < \hat{p}^1$.

COROLLARY 2. *The higher population g 's size, the higher the probability that egoism is a globally asymptotically stable state for population i .*

Proof. In Appendix B. □

6.5 Average cooperation

With n groups, average cooperation is

$$AC(n) = \sum_{g:p>\hat{p}^g} \beta^g [p + \beta^g(1-p)].$$

The results are the same as in the two-group case: differentiating with respect to β^g we get

$$\frac{\partial AC(n)}{\partial \beta^g} = p + 2\beta^g(1-p) > 0.$$

Differentiating with respect to p we get

$$\frac{\partial AC(n)}{\partial p} = \sum_{g:p>\hat{p}^g} \beta^g (1 - \beta^g) > 0.$$

Notice, however, that an increase of β^g may correspond to a decrease of a parochial group k (with $p > \hat{p}^k$) or an egosit group k' (with $p < \hat{p}^{k'}$). Therefore the effects of a variation of β^g are not clear in general. If we suppose that all groups are parochial ($p > \max \{\hat{p}^g\}$), we may call $\sum_{g:p>\hat{p}^g, g \neq i} \beta^g = 1 - \beta^i$. In this case, the results would be qualitatively similar to those in Proposition 2. If the level of strategy assortativity is not high enough ($p > \hat{p}^g$ for some t), the effect is ambiguous.

7 Discussion

We have studied the evolution of cooperation when individuals are assortatively matched according to their strategies rather than their social groups. We have considered a framework where individuals differ in their social groups and can adopt their actions (either cooperate or defect) conditional on the partner's group. As a consequence, the available strategies in the prisoner dilemma are cooperation, parochialism, anti-parochialism and egoism.

Our results have shown that cooperation or anti-parochialism are always dominated by parochialism, irrespective of the degree of strategy assortativity. Indeed, parochialists of the

same group invade cooperators and anti-parochialists, because they get the same benefits of cooperators (which are larger of the benefits of anti-parochialists) without paying the costs of cooperation in random matching.

Moreover, parochialists can invade an egoist equilibrium when strategy assortativity is sufficiently strong. This happens because, with strong assortativity, parochialists of the same group often cooperate with each other, yielding a benefit which more than offsets the higher cost sustained in random matching.

Intuitively, the average cooperation of the population increases with the level of strategy assortativity, while the relative group size plays a more articulated role according to whether parochialism is the prevailing strategy in one group or both. If only one group is parochial, which occurs for an intermediate degree of strategy assortativity, this is the small one, while the large one is egoist. As a consequence, average cooperation increases with the size of the minority group. If strategy assortativity is sufficiently high, so that parochialism emerges in both groups, then average cooperation increases with the difference in size between the two groups.

A possible limitation of the analysis is the relatively short time horizon considered, under which group sizes can be reasonably assumed to be fixed. In a longer time horizon, group size may evolve in response to average payoff of the group. Phenomena such as cultural assimilation could also be embedded in the model, leading to a change over time of the relative group size.

Our paper helps answering the following general question: is parochialism socially desirable or not? The answer depends on the counterfactual: indeed, it is worse than cooperation, and better than egoism. In our model, full cooperators are always wiped out over time, and parochialism is the best we can aim at. To obtain this result, a crucial role is played by the assumption of strategy assortativity. If we replace such an assumption with action assortativity, then full cooperation is a viable outcome, occurring for sufficiently high levels

of action assortativity, even in the presence of direct cost of interacting with an individual of a different type (Bilancini et al., 2018). This observation calls for an inquiry of the type of assortativity that is prevalent in our societies, what determines it and, from a policy perspective, how we can affect such determinants.

Another question that our results help tackling concerns the social desirability of multiculturalism in a society. As noticed by Kuran and Sandholm (2008), there is a trade-off between multiculturalism and social integration, and while the former advocates for policies designed to preserve the cultural features of some social groups or minorities, the interaction among different communities might erode specific cultural features and promote a hybridization of the population. In our setting, think of the case where the degree of strategy assortativity is intermediate, so that parochialism has established only in the minority group. Suppose the majority group is now divided into subgroups, so that strategy assortativity occurs at a finer level: each subgroup has a smaller size than the original group, and this may allow the subgroup threshold to fall below the current degree of strategy assortativity. If this happens, then the average cooperation in society increases. Suppose, instead, that the minority group is divided into subgroups. Each subgroup would still be parochial, but average cooperation would fall because, when matching occurs randomly, it is less likely to meet someone of own group. Therefore, our results suggest that sub-cultures can enhance average cooperation in large groups but not in small groups. From a different perspective, sub-cultures are more likely to enhance average cooperation when the degree of strategy assortativity is low. This is so because most groups will be made of egoists. In case of a high degree of strategy assortativity, instead, most groups will be made of parochialists, hence a finer partition of groups is likely to reduce average cooperation.

An avenue for future research concerns the investigation of the determinants of the degree of strategy assortativity. In particular, we can ask ourselves which effects are likely to be results of technological progress, as time goes on, on p . Honestly, we do not see a clear-cut

answer to the question. Telecommunications have already created significant room for remote interaction, and future developments in digital communication will further reduce the role of geographical considerations in constraining interactions. So the question becomes whether the interactions on the internet are characterized by more or less strategy assortativity than physical interactions. Public regulation of web activity can affect the answer.

A potentially interesting line for future research is related to the concept of identity ([Akerlof and Kranton, 2000](#)). If we think of assortativity as defined on identity, rather than on types or preferences/strategies, a relevant question arises regarding the determinants of identity: is the individual self-image mostly generated by the belonging to some ethnic or religious group, or rather by actual behaviors? Such a question probably requires an empirical answer, which may help understand the relative prominence, case-by-case, of type-assortativity and strategy-assortativity.

Another route to explore is related to the possibility of individuals to deceive others by strategic mimicry of other types, e.g., adopting a different dress code to conceal group membership and appear as someone different. This could be done along the lines of [Heller and Mohlin \(2019\)](#), who study how deception and preferences might co-evolve together.

Following the insights resulting from the analysis of cooperation, we would like to conclude with a comment on implications concerning migration policies. Similar models have been employed to analyze the impact of government programs in regulating immigrants' behavior (see, for instance, [Pin and Rogers, 2015](#)). A larger quota of the minority group, which may result from a higher immigration flow, may generate an increase of average cooperation. This is what happens in our model for an intermediate degree of strategy assortativity, when only the minority group is comprised of parochialists, while the majority group is comprised by egoists.

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Appendices

A Expected payoffs

In this appendix we provide details on the expected payoffs of the main text. The analysis is developed by considering n types.

If a type i adopts CC , in case of random match she gets

$$\begin{aligned}
 \pi_{CC}^i(s, r) &= \beta \left[(s_{CC}^i + s_{CD}^i) (b - c) + (s_{DC}^i + s_{DD}^i) (-c) \right] + \\
 &\quad \sum_{g=1, g \neq i}^n \beta^g \left[(s_{CC}^g + s_{CD}^g) (b - c) + (s_{DC}^g + s_{DD}^g) (-c) \right] \\
 &= \beta^i \left[(s_{CC}^i + s_{CD}^i) b - c \right] + \sum_{g=1, g \neq i}^n \beta^g \left[(s_{CC}^g + s_{CD}^g) b - c \right] \\
 &= \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - c,
 \end{aligned}$$

while in case of assortative match she gets

$$\begin{aligned}
 \pi_{CC}^i(s, a) &= \eta_{i|CC} (b - c) + (1 - \eta_{i|CC}) (b - c) \\
 &= b - c,
 \end{aligned}$$

where “ r ” and “ a ” stand for “random” and “assortative”, respectively.

If a type i adopts CD , in case of random match she gets

$$\begin{aligned}
 \pi_{CD}^i(s, r) &= \beta^i \left[(s_{CC}^i + s_{CD}^i) (b - c) + (s_{DC}^i + s_{DD}^i) (-c) \right] + \\
 &\quad \sum_{g=1, g \neq i}^n \beta^g \left[(s_{CC}^g + s_{CD}^g) b + (s_{DC}^g + s_{DD}^g) (0) \right] \\
 &= \beta^i \left[(s_{CC}^i + s_{CD}^i) b - c \right] + \sum_{g=1, g \neq i}^n \beta^g \left[(s_{CC}^g + s_{CD}^g) b \right] \\
 &= \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \beta^i c
 \end{aligned}$$

while in case of assortative match she gets

$$\begin{aligned}
 \pi_{CD}^i(s, a) &= \eta_{i|CD} (b - c) + (1 - \eta_{i|CD}) b \\
 &= b - \eta_{i|CD} c.
 \end{aligned}$$

If a type i adopts DC , in case of random match she gets

$$\begin{aligned}
\pi_{DC}^i(s, r) &= \beta^i [(s_{CC}^i + s_{CD}^i) b + (s_{DC}^i + s_{DD}^i) (0)] + \\
&\quad \sum_{g=1, g \neq i}^n \beta^g [(s_{CC}^g + s_{CD}^g) (b - c) + (s_{DC}^g + s_{DD}^g) (-c)] \\
&= \beta^i (s_{CC}^i + s_{CD}^i) b + \sum_{g=1, g \neq i}^n \beta^g [(s_{CC}^g + s_{CD}^g) b - c] \\
&= \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \sum_{g=1, g \neq i}^n \beta^g c,
\end{aligned}$$

while in case of assortative match she gets

$$\begin{aligned}
\pi_{DC}^i(s, a) &= \eta_{i|DC} (0) + (1 - \eta_{i|DC}) (-c) \\
&= - (1 - \eta_{i|DC}) c.
\end{aligned}$$

If a type i adopts DD , in case of random match she gets

$$\begin{aligned}
\pi_{DD}^i(s, r) &= \beta^i [(s_{CC}^i + s_{CD}^i) b + (s_{DC}^i + s_{DD}^i) (0)] + \\
&\quad \sum_{g=1, g \neq i}^n \beta^g [(s_{CC}^g + s_{CD}^g) b + (s_{DC}^g + s_{DD}^g) (0)] \\
&= \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b,
\end{aligned}$$

while in case of assortative match she gets

$$\pi_{DD}^i(s, a) = \eta_{i|DD} (0) + (1 - \eta_{i|DD}) (0) = 0.$$

B Proofs

The analyses below are developed for n types.

Proof of Lemma 1

Comparing the expected payoff an individual from group i gets from parochialism and co-operation, we get

$$\begin{aligned}
\pi_{CD}^i(s) - \pi_{CC}^i(s) &= \\
& p(b - \eta_{i|CD}c) + (1-p) \left[\sum_{g=1}^n \beta^g [(s_{CC}^g + s_{CD}^g)b] - \beta^i c \right] - \\
& \left[pb + (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g)b - c \right] \\
&= p(b - \eta_{i|CD}c) + (1-p) \left[\sum_{g=1}^n \beta^g [(s_{CC}^g + s_{CD}^g)b] - \beta^i c \right] - \\
& \left[p(b - c) + (1-p) \left[\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g)b - c \right] \right] \\
&= p(1 - \eta_{i|CD})c + (1-p)[1 - \beta^i]c > 0
\end{aligned}$$

Comparing the expected payoff an individual from group i gets from parochialism and anti-parochialism, we get

$$\begin{aligned}
\pi_{CD}^i(s) - \pi_{DC}^i(s) &= \\
& p(b - \eta_{i|CD}c) + (1-p) \left[\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g)b - \beta^i c \right] \\
& - \\
& (1-p) \left(\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g)b - \sum_{g=1, g \neq i}^n \beta^g c \right) - p(1 - \eta_{i|DC})c \\
&= \\
& p[(b - \eta_{i|CD}c) + (1 - \eta_{i|DC})c] + (1-p)(1 - 2\beta^i)c \\
&> 0
\end{aligned}$$

Comparing the expected payoff an individual from group i gets from parochialism and

egoism, we get

$$\begin{aligned}
\pi_{CD}^i(s) - \pi_{DD}^i(s) &= p(b - \eta_{i|CD}c) + (1-p) \left[\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \beta^i c \right] \\
&\quad - (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b \\
&= p(b - \eta_{i|CD}c) - (1-p) \beta^i c \geq 0 \text{ for } p \geq \frac{c\beta^i}{b - (\beta^i - \eta_{i|CD})c}
\end{aligned}$$

Given Lemma 2, we might focus on the case where $\eta_{i|CD} = 1$, so that

$$\hat{p}^i \equiv \frac{c\beta^i}{b - (\beta^i - 1)c}$$

Notice also that

$$\frac{c\beta^i}{b - (\beta^i - 1)c} < 1.$$

Proof of Corollaries 1 and 2

Differentiation of \hat{p}^i with respect to β^i yields

$$\frac{\partial}{\partial \beta^i} \left[\frac{c\beta^i}{b - (\beta^i - 1)c} \right] = \frac{[b - c(\beta^i - 1)]c + \beta^i c^2}{(b - c(\beta^i - 1))^2} > 0. \quad (8)$$

In case of $n = 2$, then equation (8) implies $\frac{\partial \hat{p}^i}{\partial \beta^i} < 0$.

C Inactive thresholds

LEMMA 2. *For every $s \in \text{int}(S)$, $g \in \{i, j\}$, $\pi_{DD}^g(s) > \pi_{CC}^g(s)$ for $p < \tilde{p}$, where $\tilde{p} \equiv \frac{c}{b}$.*

Proof. Comparing the expected payoff an individual from group i gets from egoism and cooperation, we get

$$\begin{aligned}
\pi_{DD}^i(s) - \pi_{CC}^i(s) &= (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \\
&\quad \left[pb + (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - c \right] \\
&= -pb + c > 0, \text{ for } p < \frac{c}{b}
\end{aligned}$$

□

LEMMA 3. For every $s \in \text{int}(S)$, $g \in \{i, j\}$, $\pi_{DD}^g(s) > \pi_{DC}^g(s)$ for $p \leq \bar{p}$, where

$$\bar{p} \equiv \frac{\sum_{g=1, g \neq i}^n \beta^g}{\sum_{g=1, g \neq i}^n \beta^g - \eta_{i|DC} + 1}$$

Proof. Comparing the expected payoff an individual from group i gets from egoism and anti-parochialism, we get

$$\begin{aligned} \pi_{DD}^i(s) - \pi_{DC}^i(s) &= (1-p) \sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \\ &\quad \left[p(1 - \eta_{i|DC})c + (1-p) \left(\sum_{g=1}^n \beta^g (s_{CC}^g + s_{CD}^g) b - \sum_{g=1, g \neq i}^n \beta^g c \right) \right] \\ &= -p(1 - \eta_{i|DC}) + (1-p) \sum_{g=1, g \neq i}^n \beta^g c \geq 0, \text{ for} \\ p &< \frac{\sum_{g=1, g \neq i}^n \beta^g}{\sum_{g=1, g \neq i}^n \beta^g - \eta_{i|DC} + 1} \end{aligned}$$

□